Dynamical Casimir effects with moving ground-state atoms: emission of photons, nonlocal and quantum Sagnac interferometric phases

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III Workshop on Nonstationary Systems

Centro Internacional de Física - UnB

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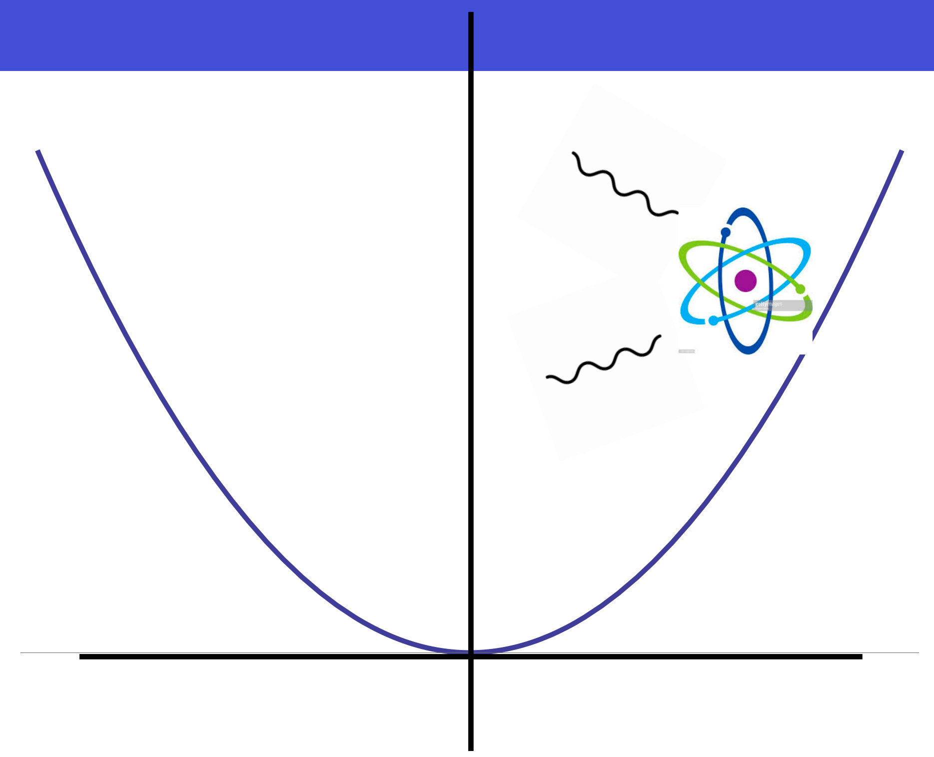
Outline

- Microscopic Dynamical Casimir Effect
- Geometric and non-local Casimir atomic phases
- Quantum Sagnac Effect

Microscopic dynamical Casimir Effect

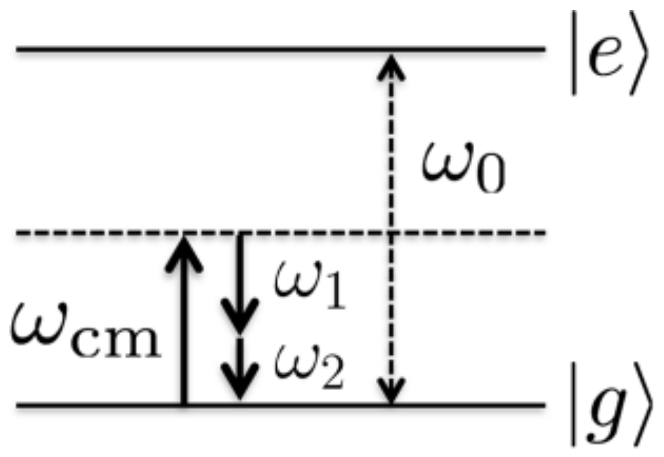
Atomic origin of the DCE?

consider an atom in a potential well, frequency ω_{cm}

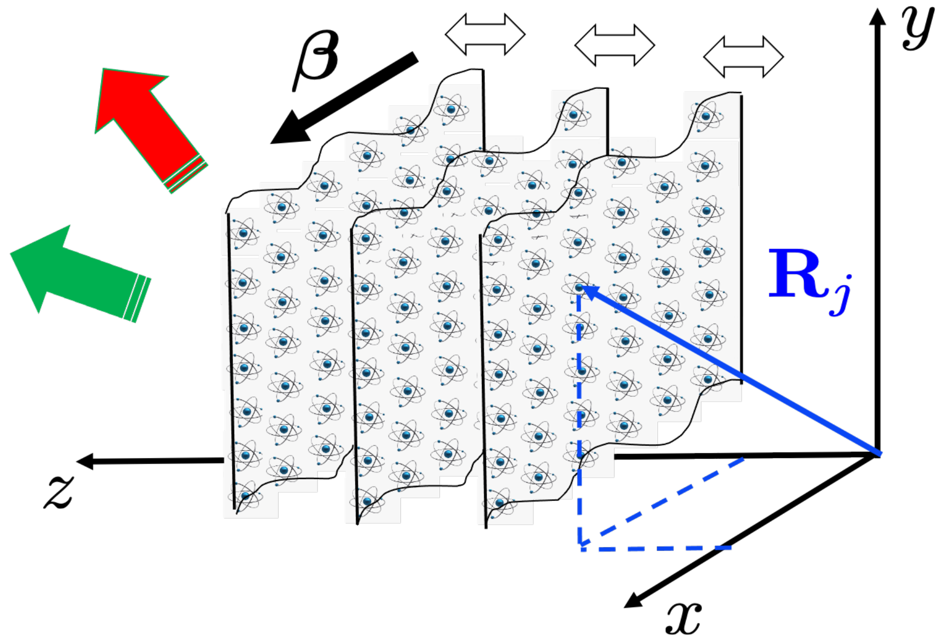


➡ Microscopic dynamical Casimir effect

Internal degrees of freedom are quantum and define energy levels



collection of atoms, spatio-temporal modulations: Dalvit & Kort-Kamp 2021



atom+surface: Belen-Farias et al 2019; Fosco, Lombardo & Mazzitelli 2021

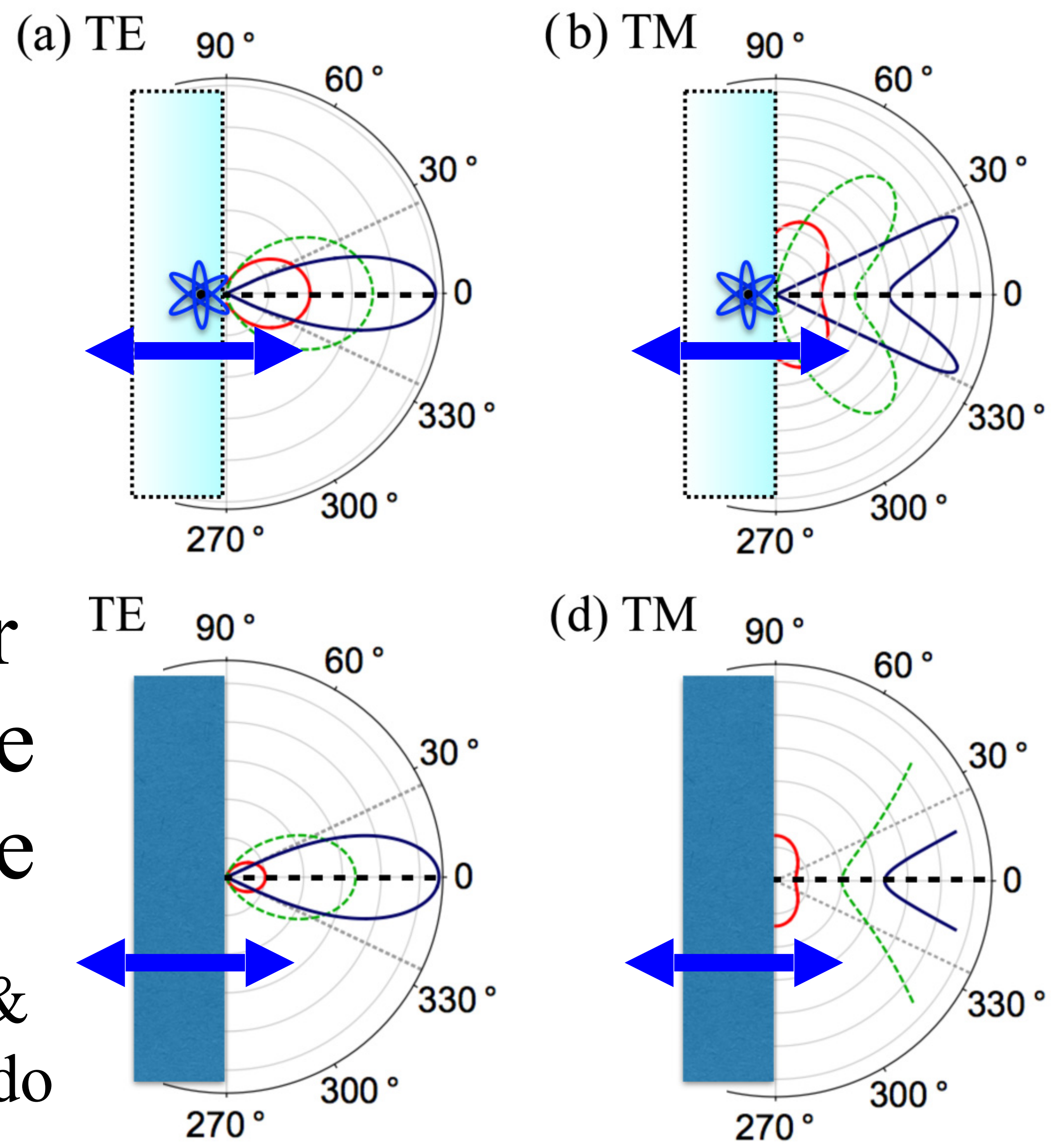
Angular spectra: comparison with material surface

Melo e Souza, Impens & MN 2018

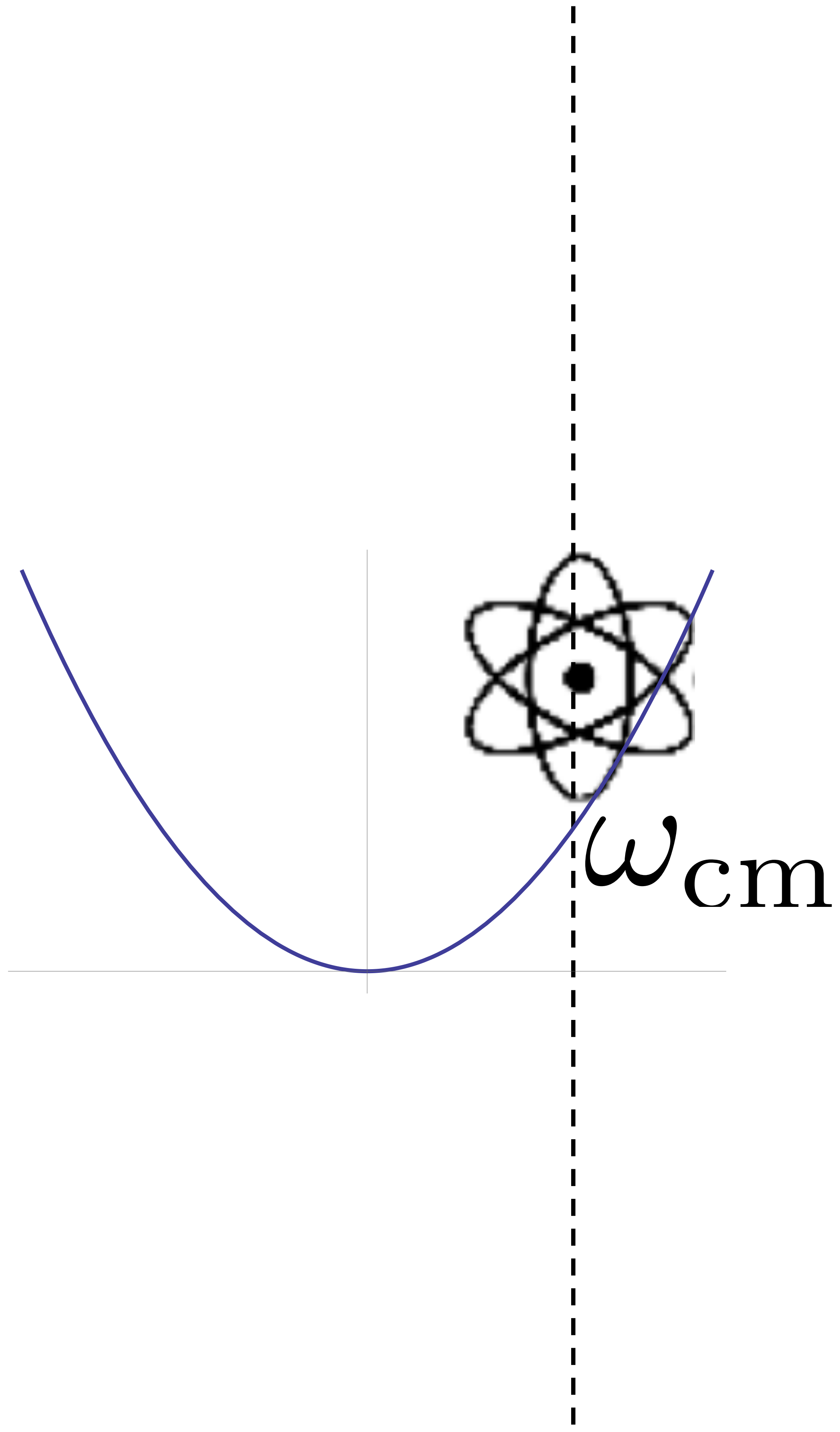
atom

planar infinite surface

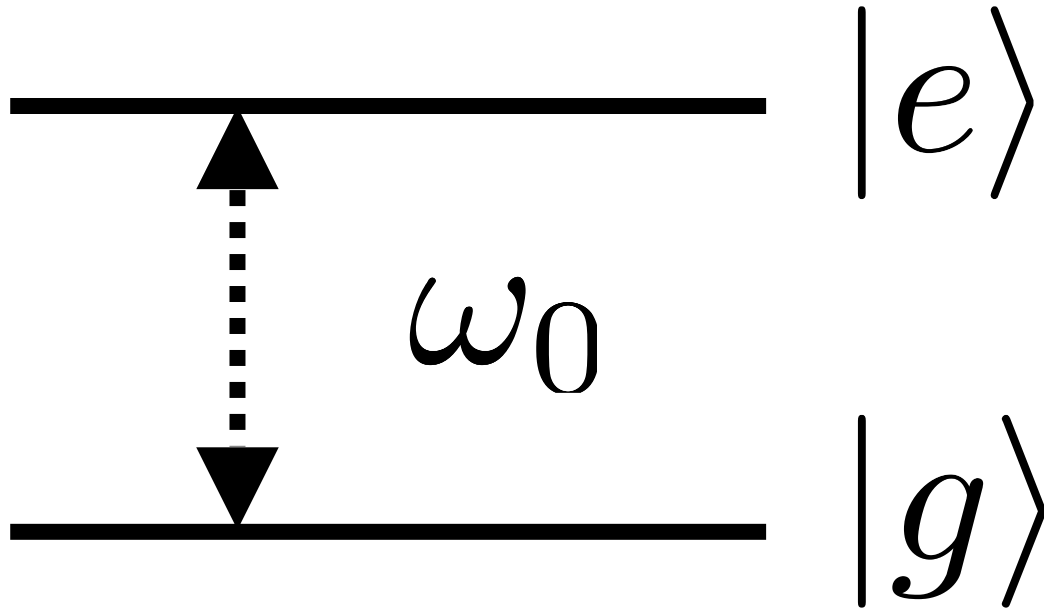
PAMN & L Machado 1996



Microscopic dynamical Casimir effect: model



Two-level atom:



set in prescribed harmonic motion:

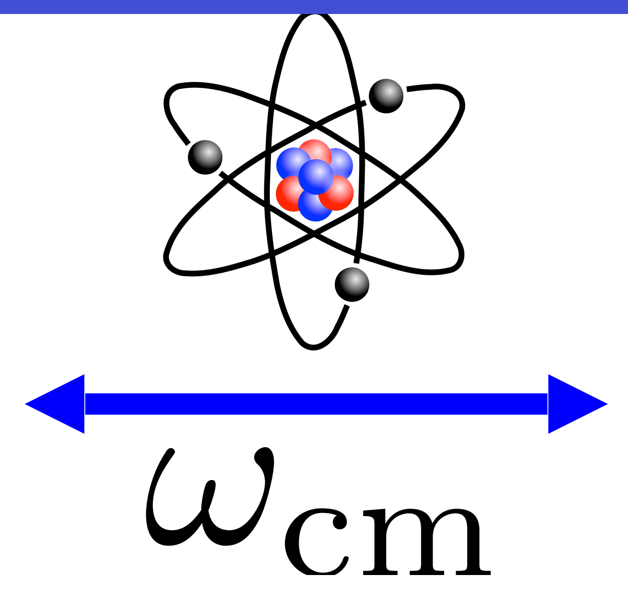
$$\mathbf{r}(t) = \mathbf{a}\cos(\omega_{\rm cm} t)$$

Classical treatment
of the center-of-mass atomic position

Atom initially in ground state

Microscopic dynamical Casimir effect: model

Oscillating two-level atom

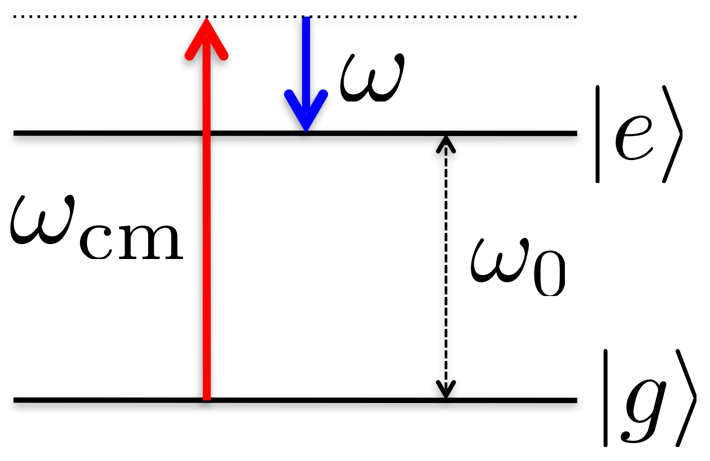


Two regimes

$\omega_{\rm cm} > \omega_0$

Motion-induced excitation

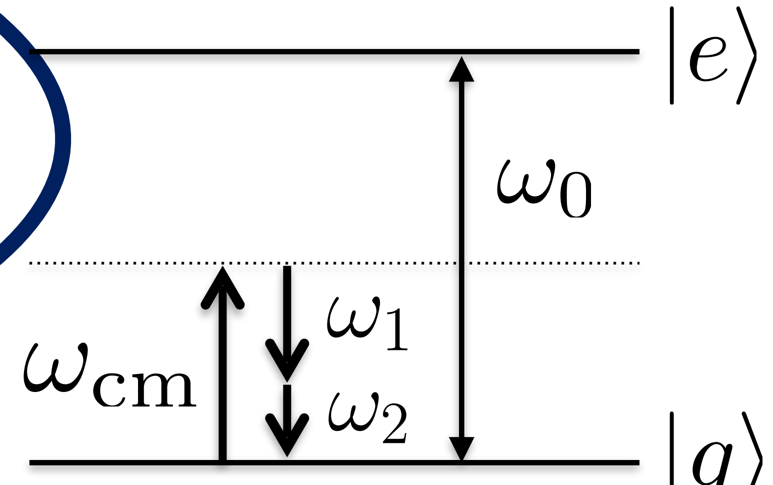
One-photon process



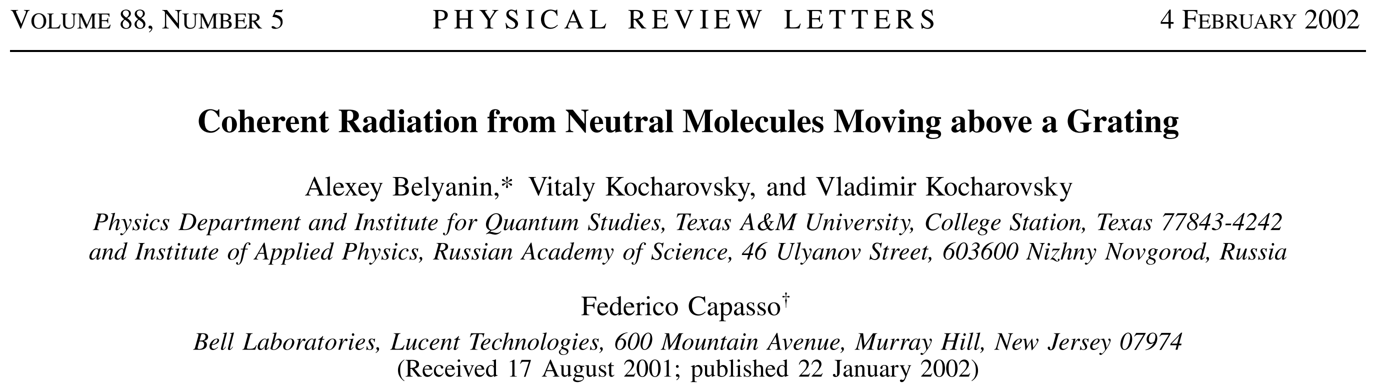
$\omega_{\rm cm} < \omega_0$

Microscopic Dynamical Casimir Effect

Two-photon process



Related problem: molecule moving on top of a grating



VOLUME 88, NUMBER 5 PHYSICAL REVIEW LETTERS 4 FEBRUARY 2002

Coherent Radiation from Neutral Molecules Moving above a Grating

Alexey Belyanin,* Vitaly Kocharovsky, and Vladimir Kocharovsky
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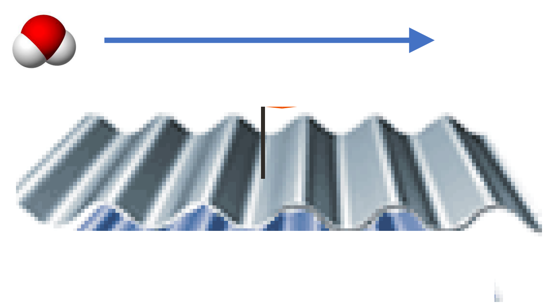
(Received 17 August 2001; published 22 January 2002)



Microscopic dynamical Casimir effect: model

Dipole interaction for an atom at rest:

$$\hat{V}(\mathbf{r}(t)) = -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\mathbf{r}(t))$$

Dipole operator

Electric field operator

Average atomic position

$$\mathbf{r}(t) = \langle \hat{\mathbf{r}} \rangle (t)$$

External velocity

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt}$$

For a moving atom: electric field in the comoving frame

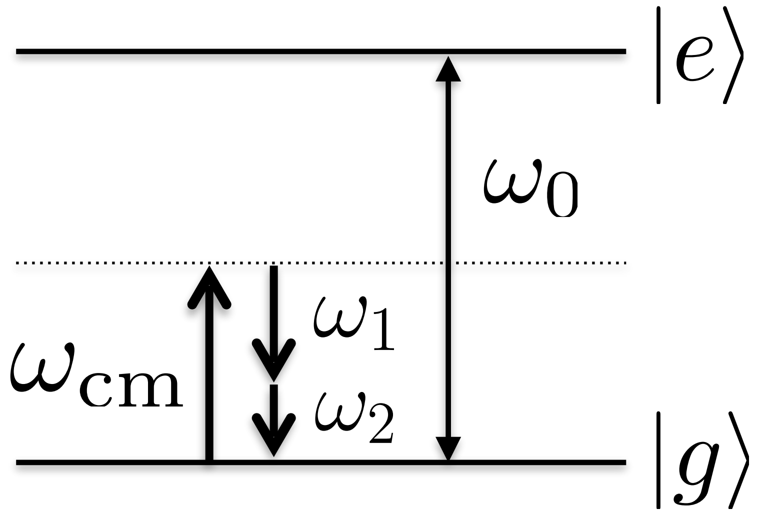
$$\hat{\mathbf{E}}'(\mathbf{r}(t)) = \hat{\mathbf{E}}(\mathbf{r}(t)) + \frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}(\mathbf{r}(t))$$

Dipolar interaction for a moving atom:

$$\hat{V}_R(\mathbf{r}(t)) = \hat{V}(\mathbf{r}(t)) \underbrace{- \hat{\mathbf{d}} \cdot \mathbf{v}(t) \times \hat{\mathbf{B}}(\mathbf{r}(t))}_{\text{Röntgen term}}$$

Baxter, Babiker & Loudon 1993; Wilkens 1994

Microscopic dynamical Casimir effect: model



Initial quantum state: $|\Psi(0)\rangle = |g\rangle \otimes |0\rangle$

How to describe the MDCE photon pair production?

Use 2nd-order perturbation with

$$\hat{V}_R(\mathbf{r}(t)) = -\,\hat{\mathbf{d}} \cdot \Big(\hat{E}(\mathbf{r}(t) + \mathbf{v}(t) \times \hat{\mathbf{B}}(\mathbf{r}(t) \Big)$$

Dalvit & Kort-Kamp 2021

Use 1st order perturbation with an effective field Hamiltonian [Passante, Power, Thirunamachandran, 1998]

$$\hat{H}_{\rm eff}(\mathbf{r}(t)) = -\frac{\alpha(0)}{2}\hat{E}'(\mathbf{r}(t))^2$$

$$\hat{\mathbf{E}}'(\mathbf{r}(t)) = \hat{\mathbf{E}}(\mathbf{r}(t)) + \frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}(\mathbf{r}(t))$$

ground state polarizability $\alpha(\omega_{\mathbf{k}}) \simeq \alpha(0)$

Effective Hamiltonians from a systematic approach
A. S. Santos, P. H. Pereira, P. P. Abrantes, C. Farina, PAMN, and R. de Melo e Souza, Entropy 2024.

Microscopic dynamical Casimir effect: model

$$\hat{H}_{\text{eff}}(\mathbf{r}(t)) = -\frac{\alpha(0)}{2}\hat{E}'(\mathbf{r}(t))^2$$

$$\hat{\mathbf{E}}'(\mathbf{r}(t)) = \hat{\mathbf{E}}(\mathbf{r}(t)) + \frac{\mathbf{v}(t)}{c} \times \hat{\mathbf{B}}(\mathbf{r}(t))$$

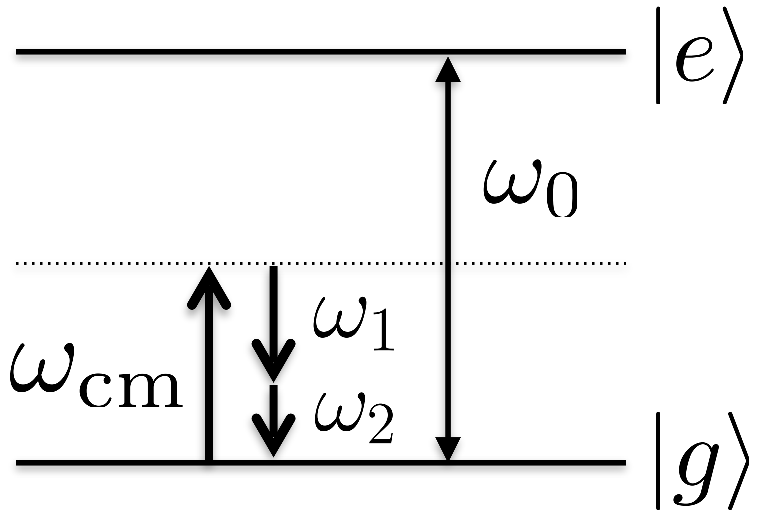
Quadratic in the field operators $\Longrightarrow$ creation of photon pairs

Field state (first-order perturbation):

$$|\psi(t)\rangle = |0\rangle + \sum_{\mathbf{k}_1\lambda_1\mathbf{k}_2\lambda_2} c_{\mathbf{k}_1\lambda_1\mathbf{k}_2\lambda_2}(t)|1_{\mathbf{k}_1\lambda_1}1_{\mathbf{k}_2\lambda_2}\rangle$$

Time-dependent perturbation theory/Fermi golden rule

$$\omega_{\text{cm}} = \omega_1 + \omega_2$$



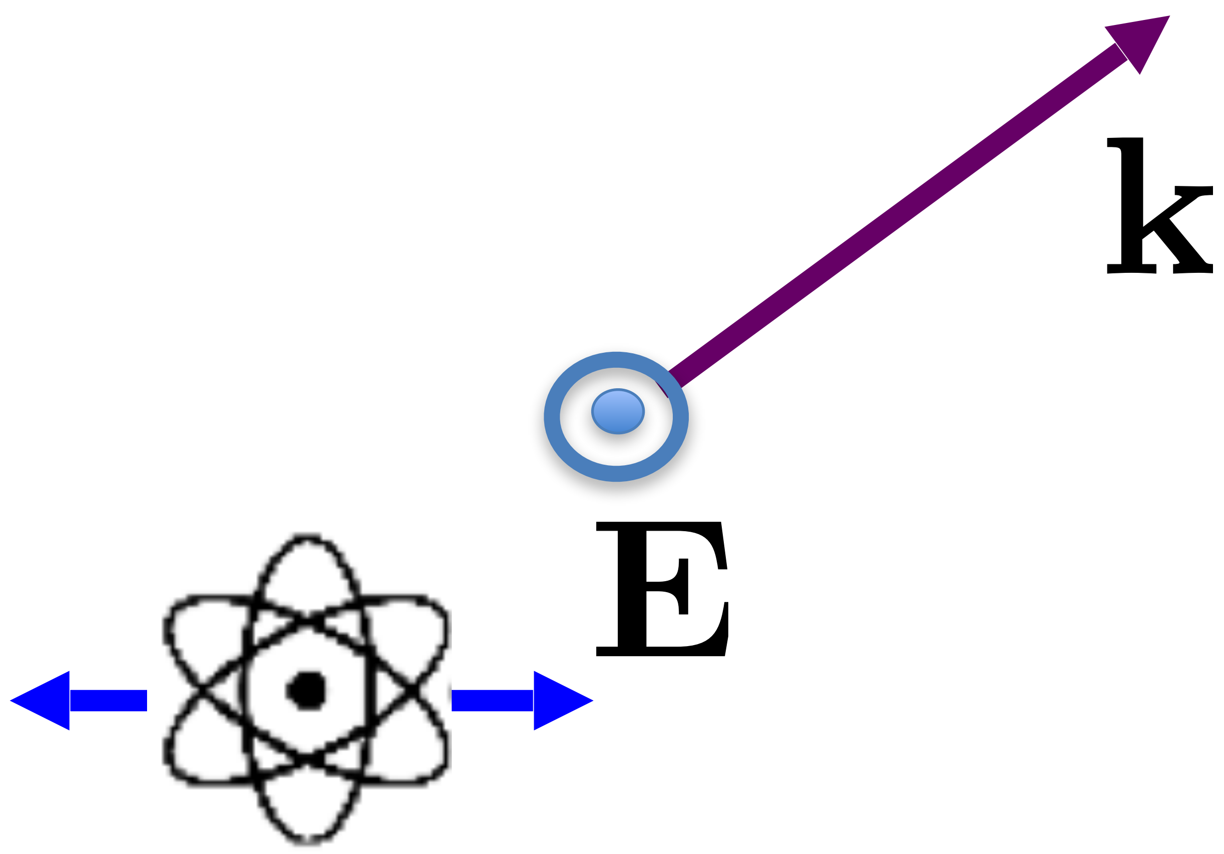
Microscopic dynamical Casimir effect: model

Probability of emission obtained from $|\langle 1_{\mathbf{k_1}\lambda_1} 1_{\mathbf{k_2}\lambda_2} | \hat{H}_{\mathrm{eff}}(\mathbf{r}(t), t) | 0 \rangle|^2$

Probability to detect a photon along a given direction/polarization:

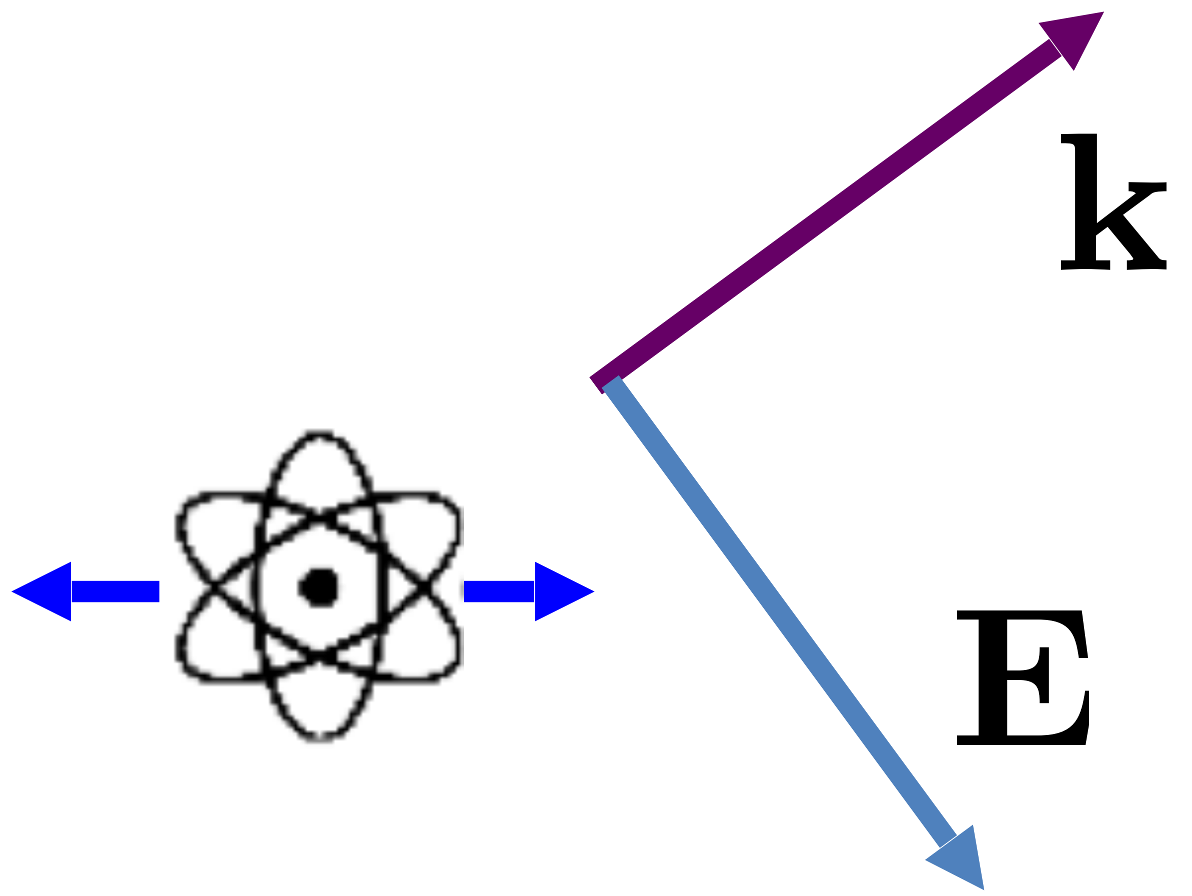
sum over all possible idle photons!

Transverse Electric (TE)



Oscillation along $\mathbf{n}$

Transverse Magnetic (TM)

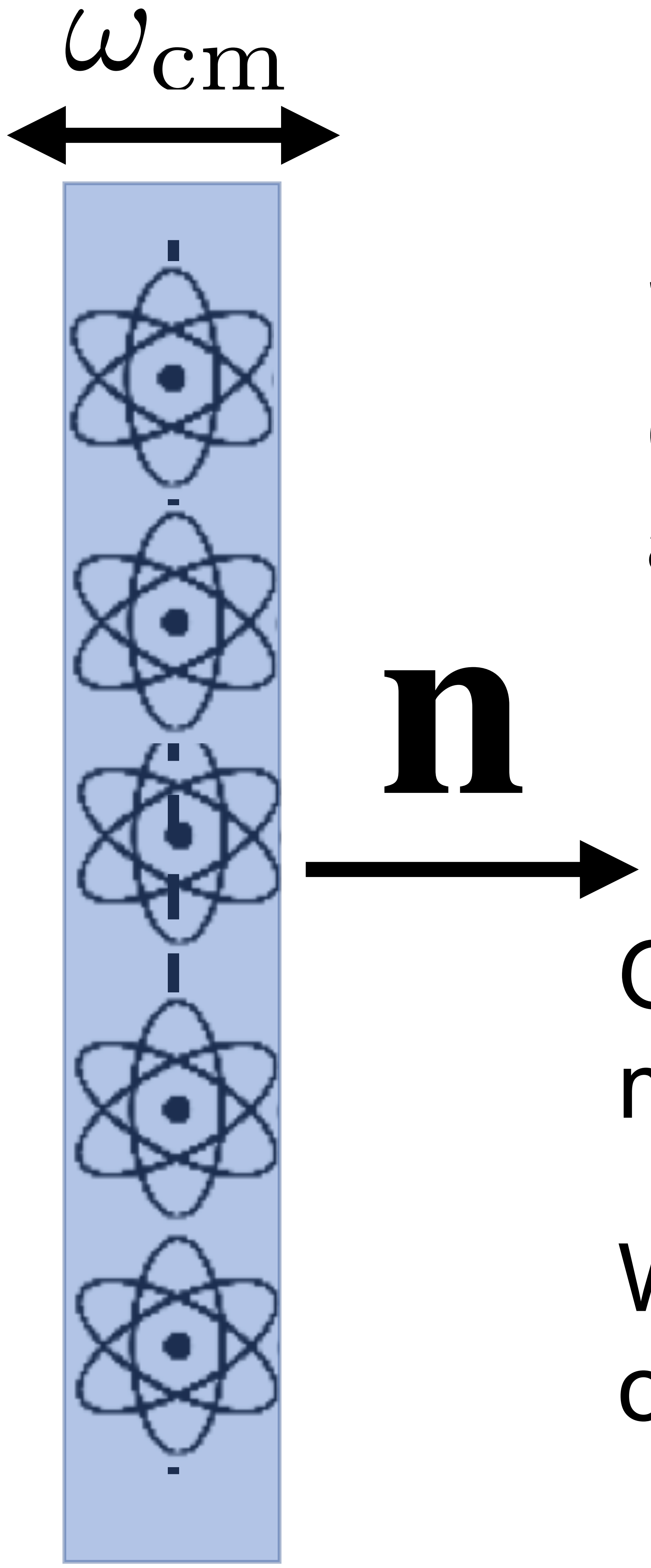


Oscillation along $\mathbf{n}$

Reference plane defined by the vectors $(\mathbf{k}, \mathbf{n})$

Microscopic dynamical Casimir effect

Microscopic vs Macroscopic Dynamical Casimir Effect



Sum contribution from a macroscopic collection of atoms:

Constructive interference condition for a quasi continuous array of atoms with identical oscillations:

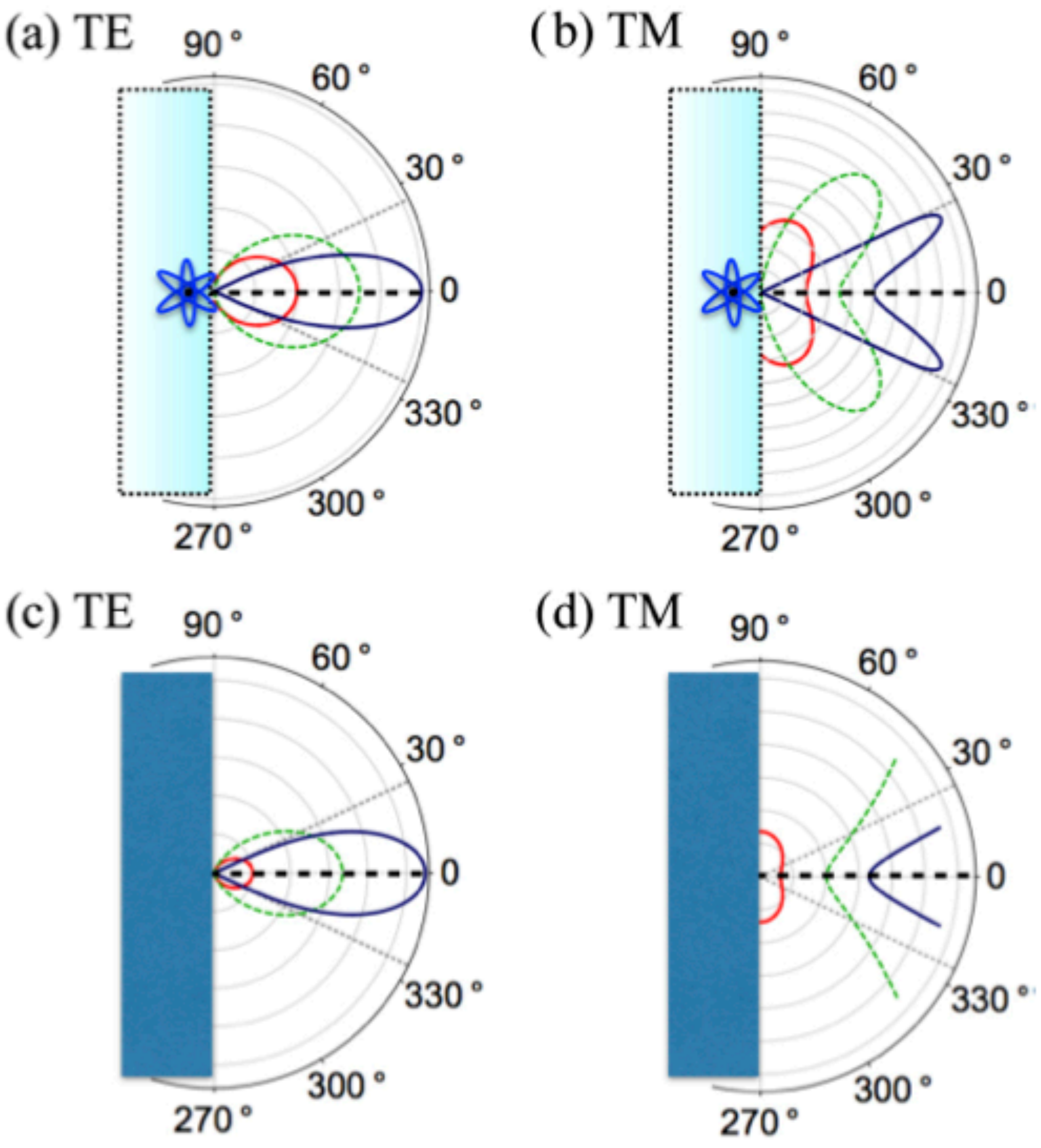
$$(\mathbf{k}_1 - \mathbf{k}_2) \times \mathbf{n} = 0$$

Only 2-photon modes that fulfill this condition of transverse momentum conservation contribute significantly.

We “impose” this condition to compare the prediction of our microscopic model with macroscopic results.

Microscopic dynamical Casimir effect

Angular spectra of atom/mirror:



$\omega = 0.3\omega_{\mathrm{cm}}, 0.5\omega_{\mathrm{cm}}, 0.7\omega_{\mathrm{cm}}$

Microscopic DCE

R. M. Souza, F Impens, PAMN, Phys Rev. A (2018).

D Dalvit, W Kort-Kamp, Universe (2021).

Macroscopic DCE

PAMN, L. Machado, Phys Rev. A (1996).

Total photon emission rate

$$\alpha(0) = 4\pi\epsilon_0 a_0^3 \qquad v_{\max} = \omega_{\mathrm{cm}} r_{\max}$$

$$\frac{dN}{dt} = \frac{23}{5670\pi}\left(\frac{a_0}{r_{\max}}\right)^6 \left(\frac{v_{\max}}{c}\right)^8 \omega_{\mathrm{cm}}$$

Look for 'dynamical Casimir - like' effects with atom interferometers probing the Casimir-Polder interaction with a surface…

Outline

- Microscopic Dynamical Casimir Effect
- Geometric and non-local Casimir atomic phases
- Quantum Sagnac Effect

introduction: atom interferometers

PRL **95,** 133201 (2005) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2005

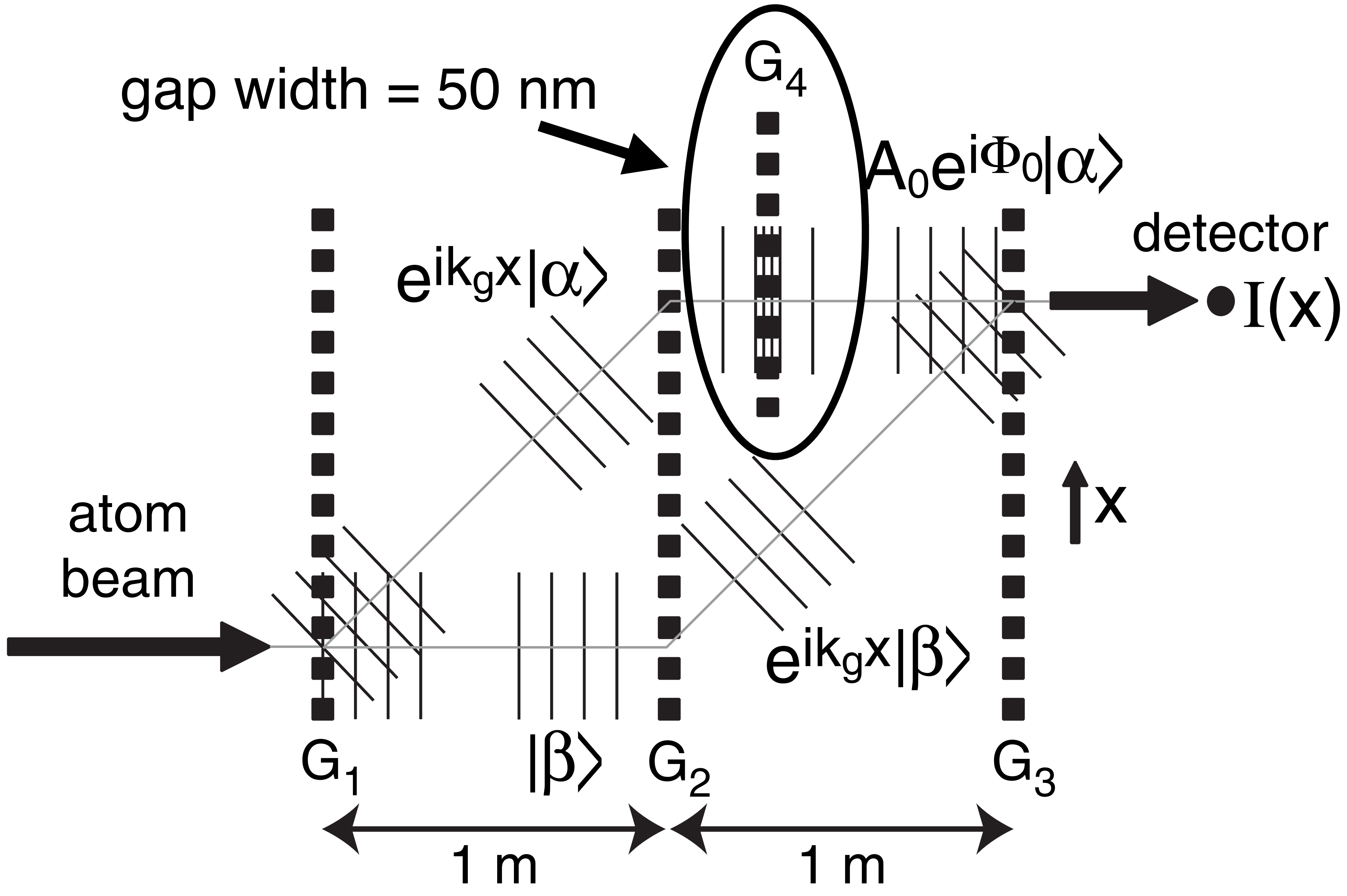
Observation of Atom Wave Phase Shifts Induced by Van Der Waals Atom-Surface Interactions

John D. Perreault and Alexander D. Cronin
University of Arizona, Tucson, Arizona 85721, USA

In both paths, atom remains in the internal ground state

Atom-Surface interaction in the nano grating

Shift of the atomic fringes



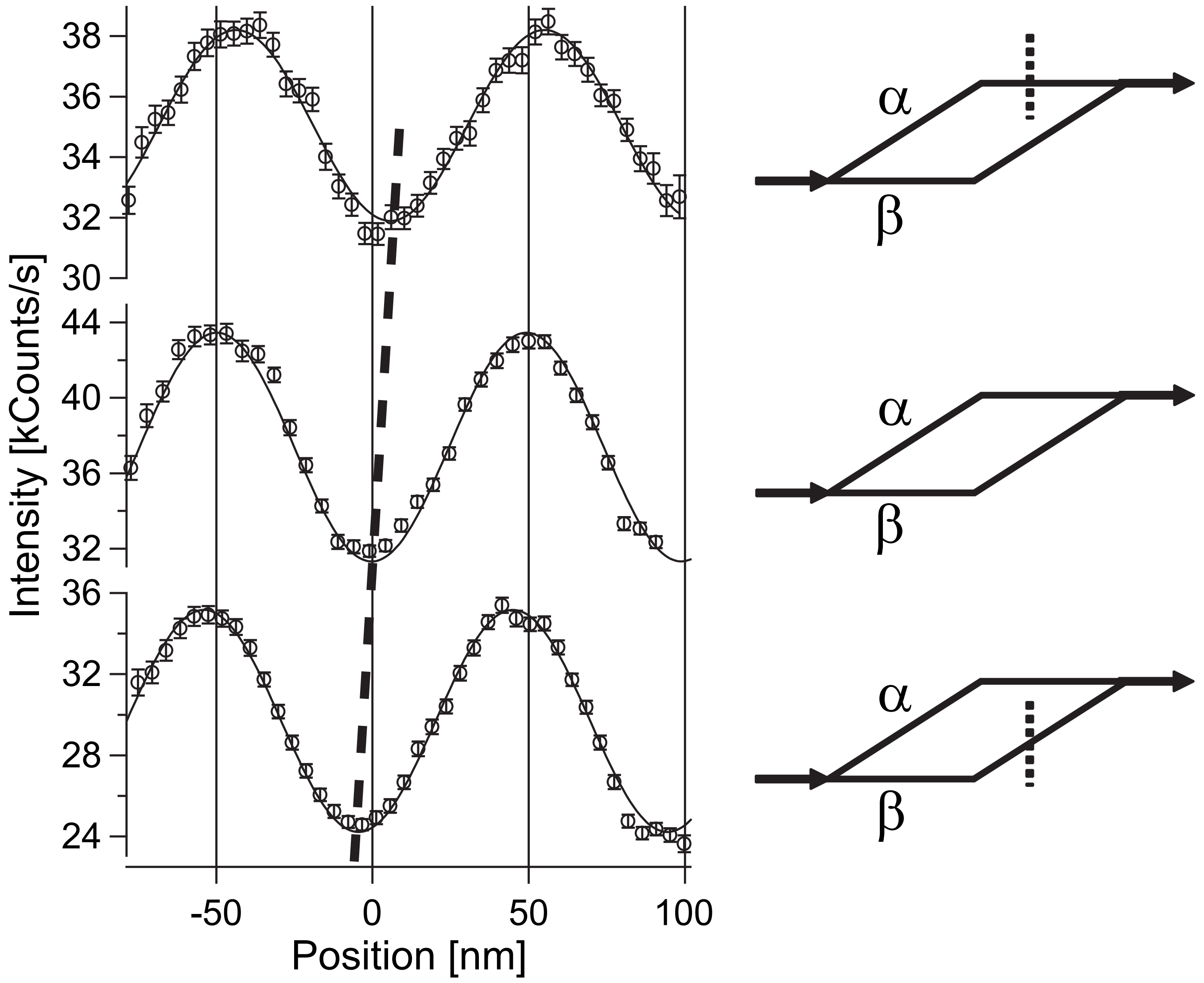
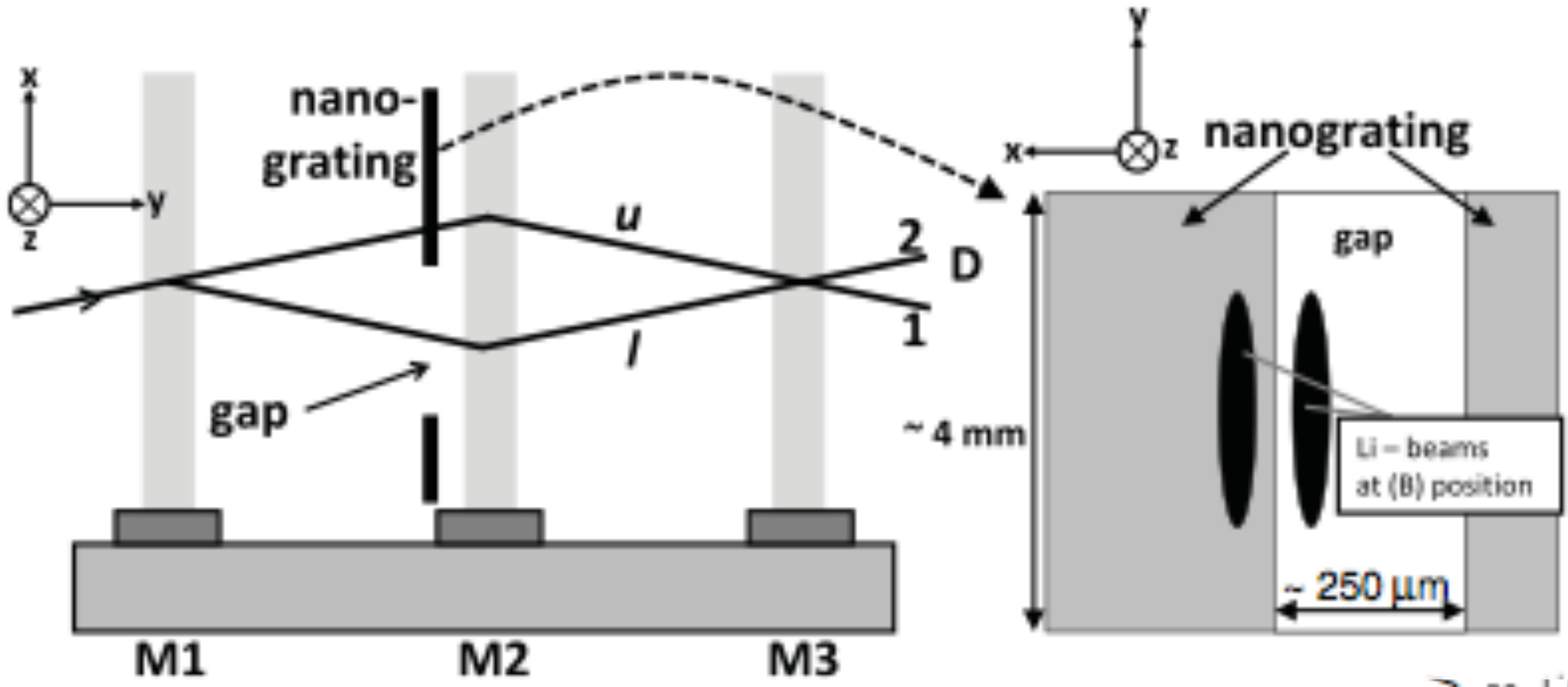
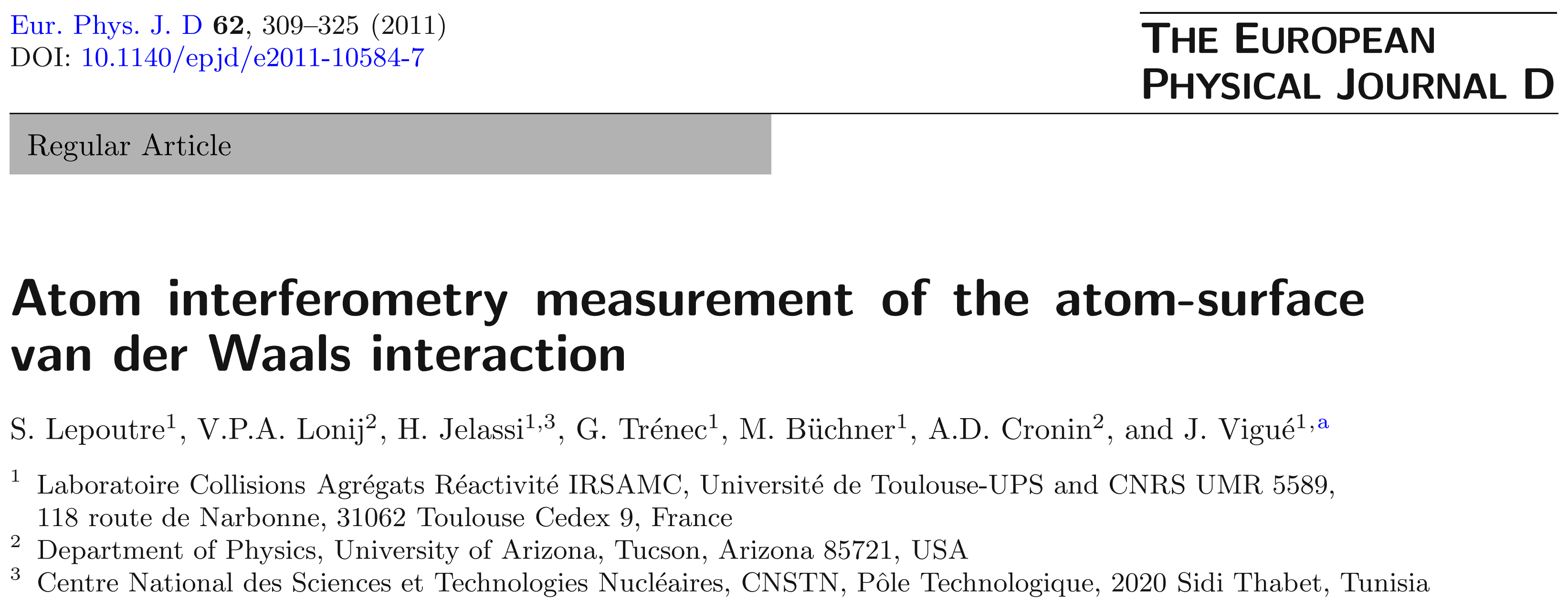


FIG. 3. Interference pattern observed when the grating G_4 is inserted into path α or β of the atom interferometer. Each interference pattern represents 5 s of data. The intensity error bars are arrived at by assuming Poisson statistics for the number of detected atoms. The dashed line in the plots is a visual aid to help illustrate the measured phase shift of 0.3 rad. Notice how the phase shift induced by placing G_4 in path α or β has opposite sign. The sign of the phase shift is also consistent with the atom experiencing an attractive potential as it passes through G_4.

introduction: atom interferometers

Bragg atom interferometer





Eur. Phys. J. D **62**, 309–325 (2011)
DOI: 10.1140/epjd/e2011-10584-7

THE EUROPEAN
PHYSICAL JOURNAL D

Regular Article

Atom interferometry measurement of the atom-surface van der Waals interaction

S. Lepoutre[1], V.P.A. Lonij[2], H. Jelassi[1,3], G. Trénec[1], M. Büchner[1], A.D. Cronin[2], and J. Vigué[1,a]

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[2] Department of Physics, University of Arizona, Tucson, Arizona 85721, USA
[3] Centre National des Sciences et Technologies Nucléaires, CNSTN, Pôle Technologique, 2020 Sidi Thabet, Tunisia

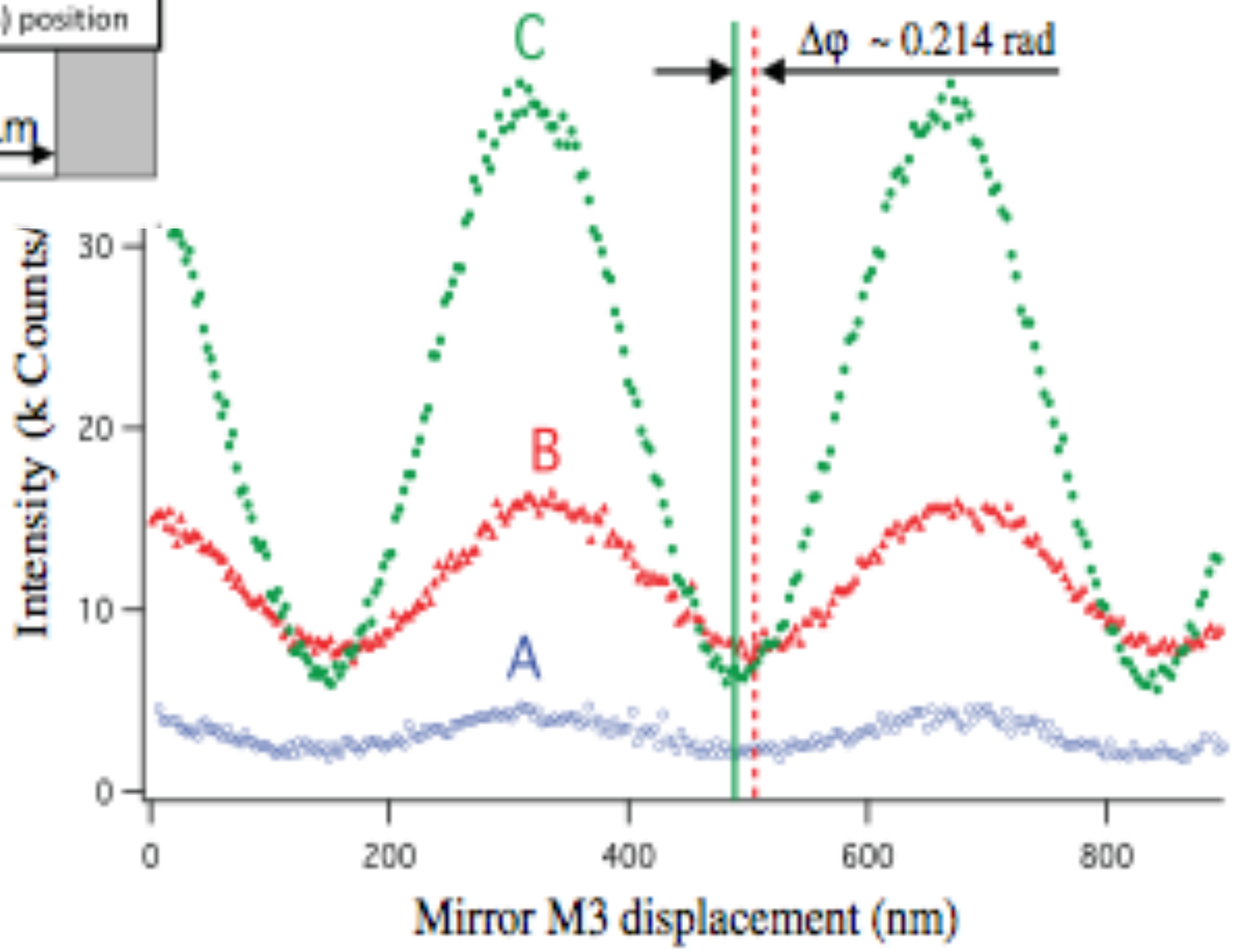
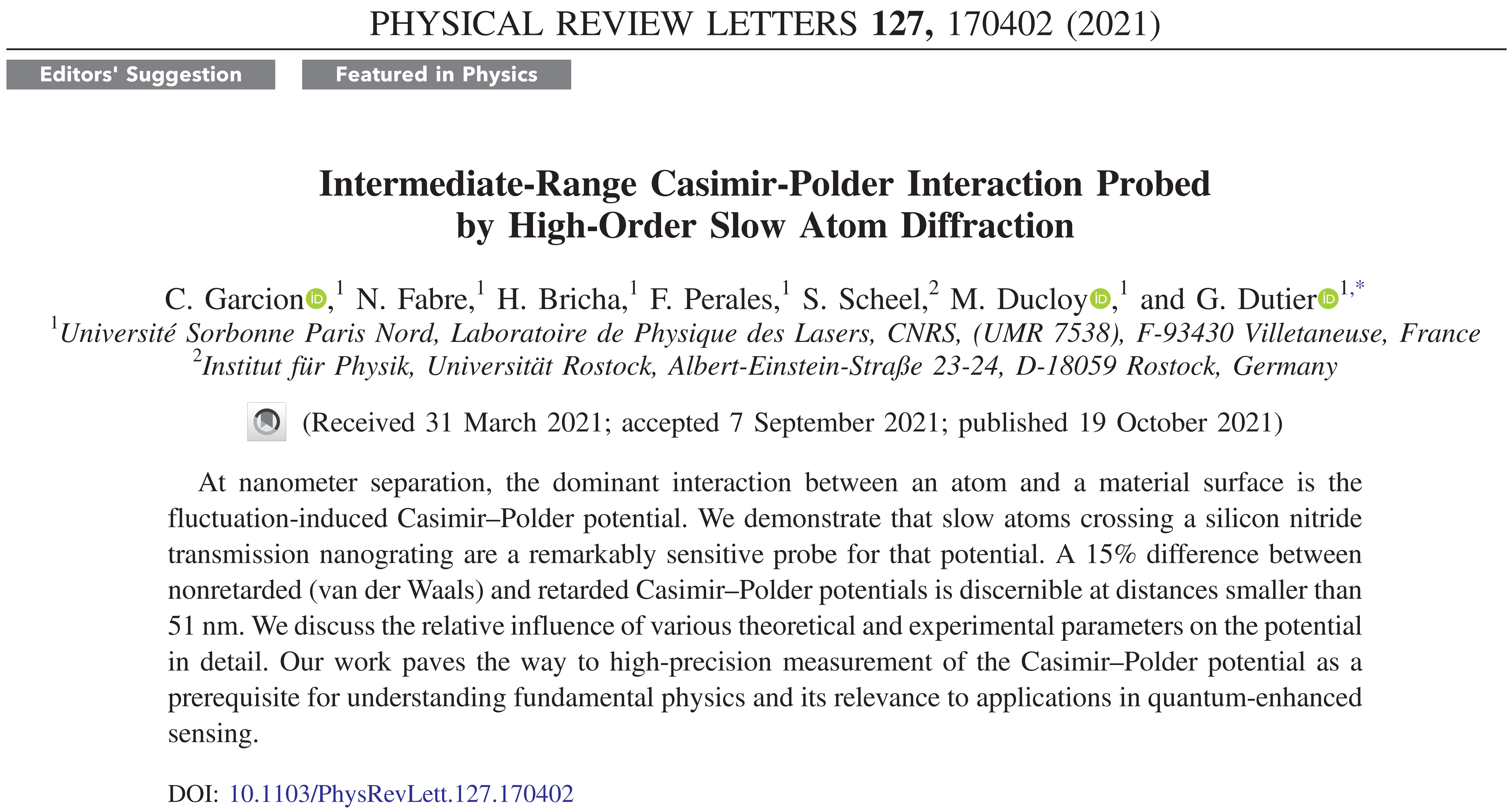


Fig. 2: (Colour on-line) Atom interference fringes recorded with (A) both arms (visibility $\mathcal{V}_A = 32\%$), (B) one arm ($\mathcal{V}_B = 34\%$), or (C) neither arm ($\mathcal{V}_C = 72\%$) passing through the nano-structure, with a lithium beam velocity $v = 1062 \pm 20$ m/s. The counting period is 0.1 s per data point.

John D. Perreault and Alexander D. Cronin, PRL **95**, 133201 (2005)
S. Lepoutre, H. Jelassi, V.P.A. Lonig, G. Trénec, M. Büchner, A. D. Cronin, and J. Vigué, EPL **88**, 20002 (2009)
S. Lepoutre et al. , EPJD **62**, 309 (2011)

introduction: atom interferometers



PHYSICAL REVIEW LETTERS **127,** 170402 (2021)

Editors' Suggestion **Featured in Physics**

Intermediate-Range Casimir-Polder Interaction Probed by High-Order Slow Atom Diffraction

C. Garcion,[1] N. Fabre,[1] H. Bricha,[1] F. Perales,[1] S. Scheel,[2] M. Ducloy,[1] and G. Dutier[1,*]

[1]*Université Sorbonne Paris Nord, Laboratoire de Physique des Lasers, CNRS, (UMR 7538), F-93430 Villetaneuse, France*

[2]*Institut für Physik, Universität Rostock, Albert-Einstein-Straße 23-24, D-18059 Rostock, Germany*

(Received 31 March 2021; accepted 7 September 2021; published 19 October 2021)

At nanometer separation, the dominant interaction between an atom and a material surface is the fluctuation-induced Casimir–Polder potential. We demonstrate that slow atoms crossing a silicon nitride transmission nanograting are a remarkably sensitive probe for that potential. A 15% difference between nonretarded (van der Waals) and retarded Casimir–Polder potentials is discernible at distances smaller than 51 nm. We discuss the relative influence of various theoretical and experimental parameters on the potential in detail. Our work paves the way to high-precision measurement of the Casimir–Polder potential as a prerequisite for understanding fundamental physics and its relevance to applications in quantum-enhanced sensing.

DOI: 10.1103/PhysRevLett.127.170402

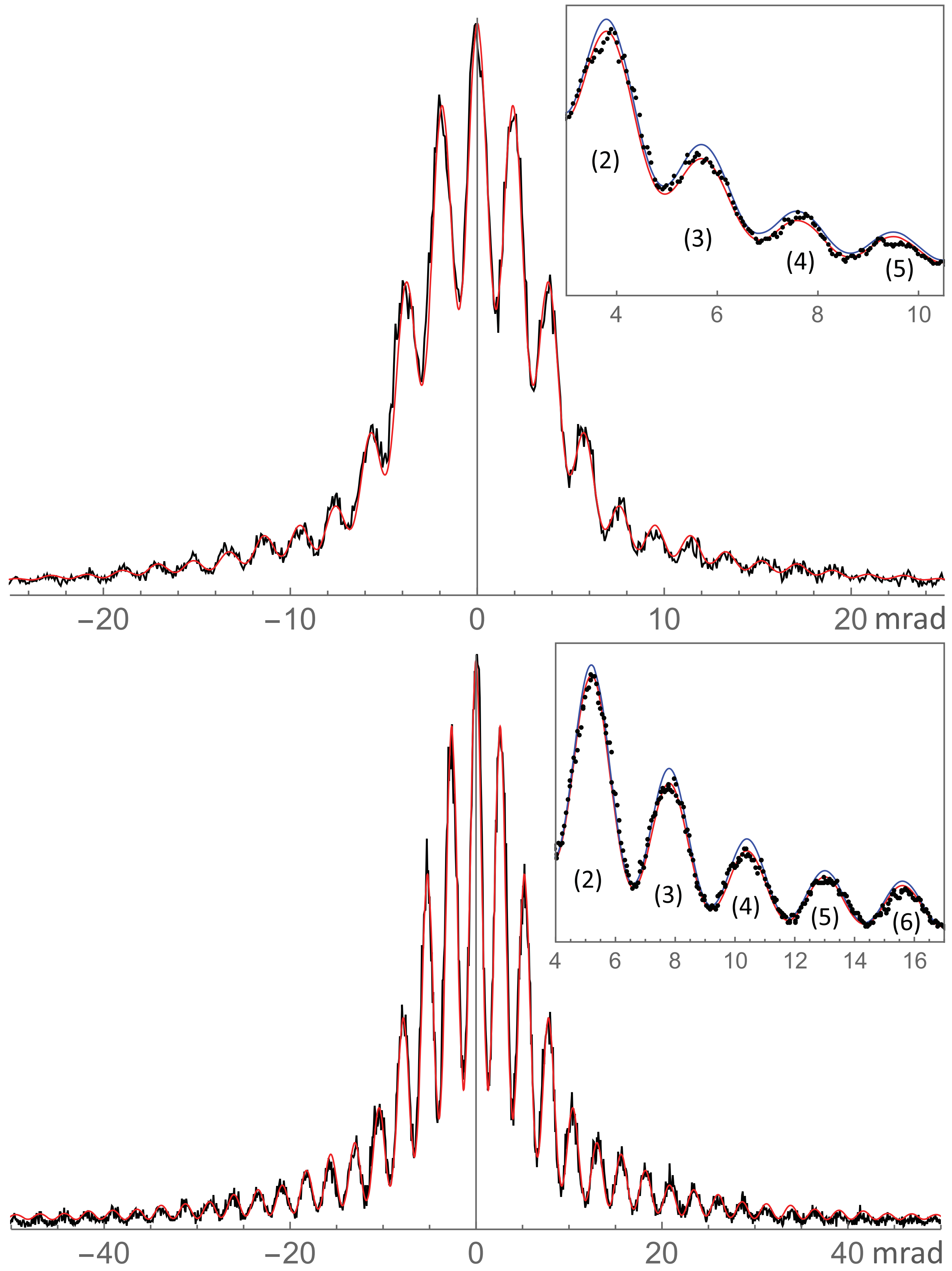
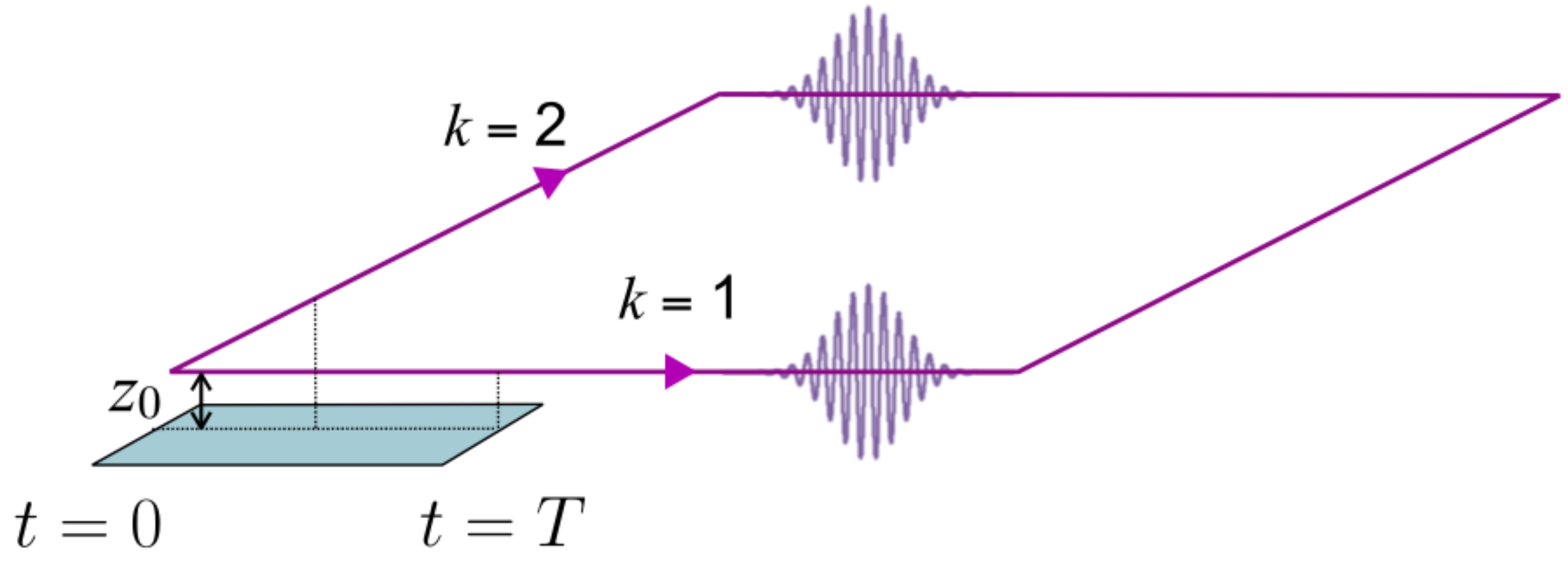


FIG. 2. Experimental diffraction spectra for beam velocities of 26 ms^{-1} (top) and 19.1 ms^{-1} (bottom) in black. The red curves are theoretical spectra with a single adjustable parameter (d_0). The insets show individual diffraction orders. Black dots result from experimental spectra averaged over positive and negative diffraction orders. Red (blue) curves are calculated with CP (vdW) potentials.

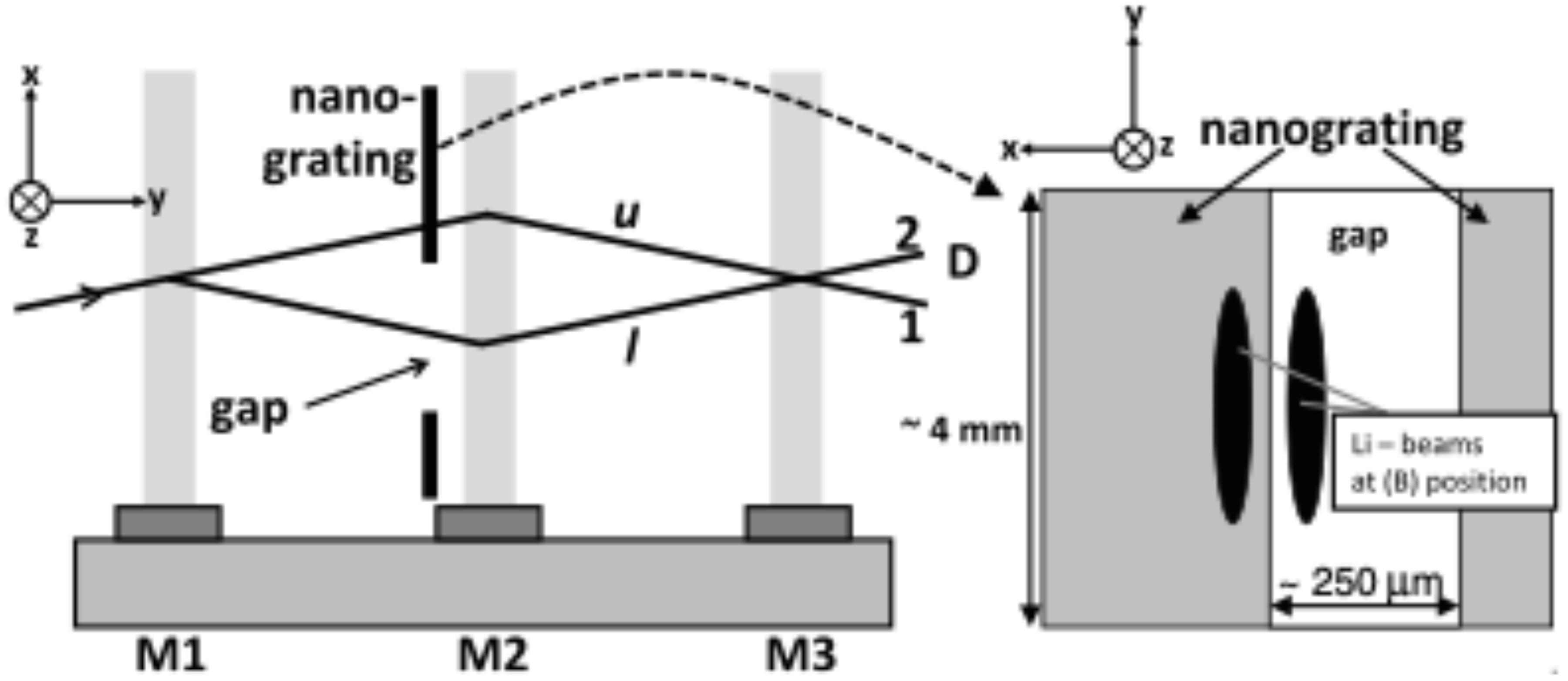
Casimir atom interferometry in the quasi-static limit



Casimir atomic phase in the quasi-static limit

$$\phi^{\mathrm{qs}} = -\frac{1}{\hbar}\int_0^T dt\, U_{\mathrm{vdW}}(\mathbf{r}(t))$$

Dispersive potential
(e.g. van der Waals potential)



John D. Perreault and Alexander D. Cronin, PRL **95,** 133201 (2005);
S. Lepoutre et al., EPL **88**, 20002 (2009); S. Lepoutre et al. , EPJD **62**, 309 (2011)

local Casimir atomic phase

Quasi-static Casimir phase: $\phi^{\mathrm{qs}} = -\frac{1}{\hbar}\int_0^T dt\, U_{\mathrm{vdW}}(\mathbf{r}(t))$

Full Casimir phase (including atomic motion): $\phi = -\frac{1}{\hbar}\int_0^T dt\, \overline{U}_{\mathrm{vdW}}(\mathbf{r}(t))$

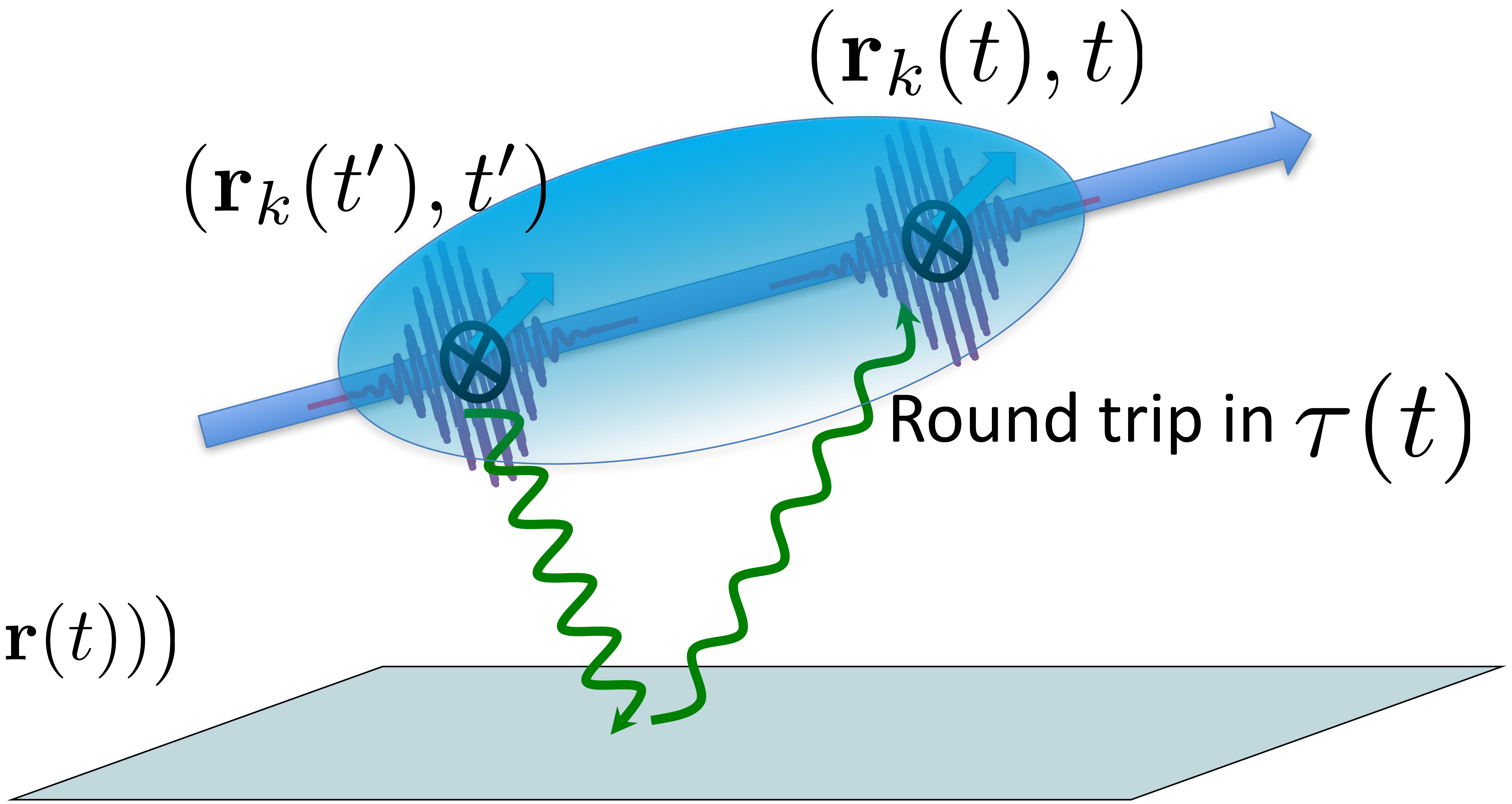
Coarse-Grained Potential: $\overline{U}_{\mathrm{vdW}}(\mathbf{r}(t)) = \frac{1}{\tau(t)}\int_t^{t+\tau(t)} dt' U_{\mathrm{vdW}}(\mathbf{r}(t'))$

$\tau(t)$ Virtual photon exchange duration

All atomic positions during the photon exchange taken into account!

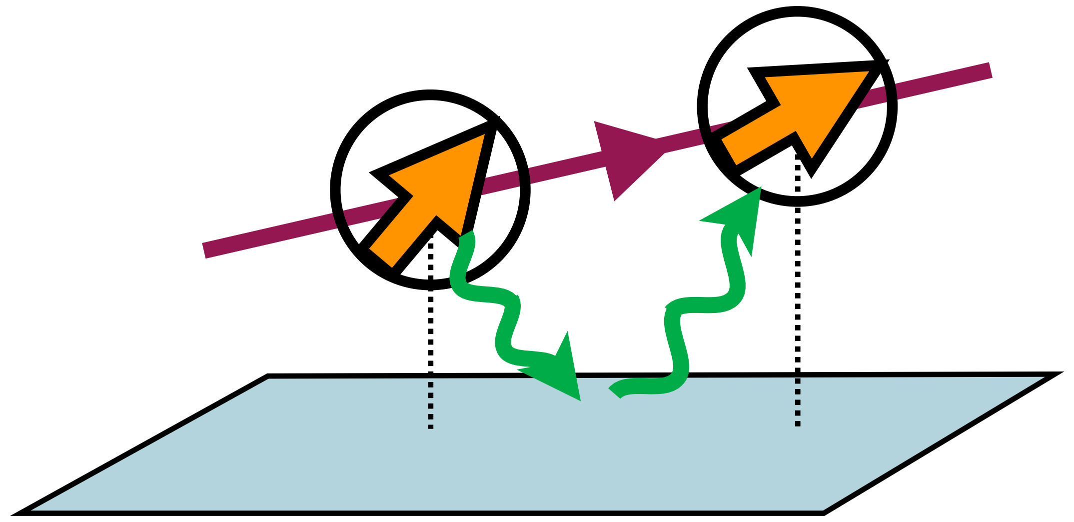
Local *Dynamical Casimir-like* phase:

$$\phi^{\mathrm{mot}} = -\frac{1}{\hbar}\int_0^T dt\, \left(\overline{U}_{\mathrm{vdW}}(\mathbf{r}(t)) - U_{\mathrm{vdW}}(\mathbf{r}(t))\right)$$

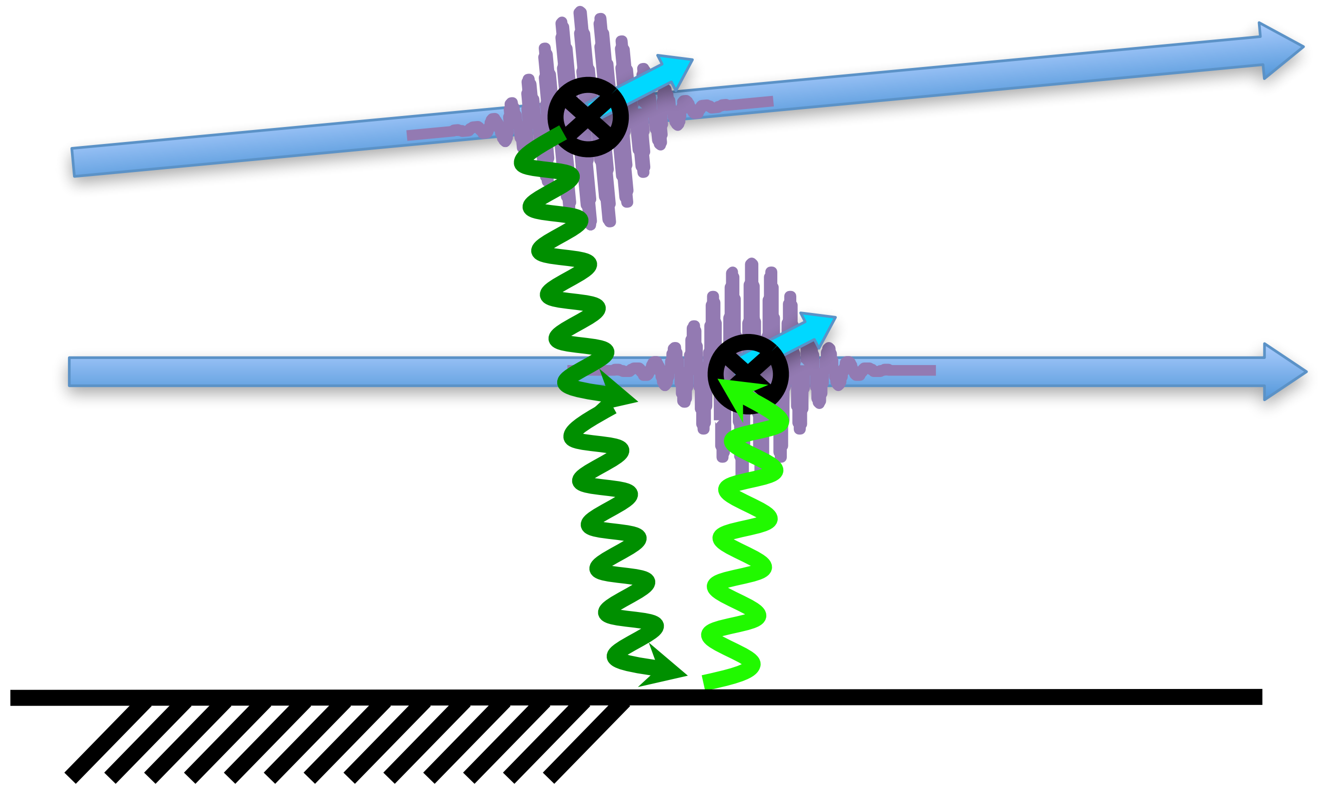


non-local Casimir atomic phase

atom-surface van der Waals interaction:
fluctuating dipole interacts with its **own field**, after reflection by surface



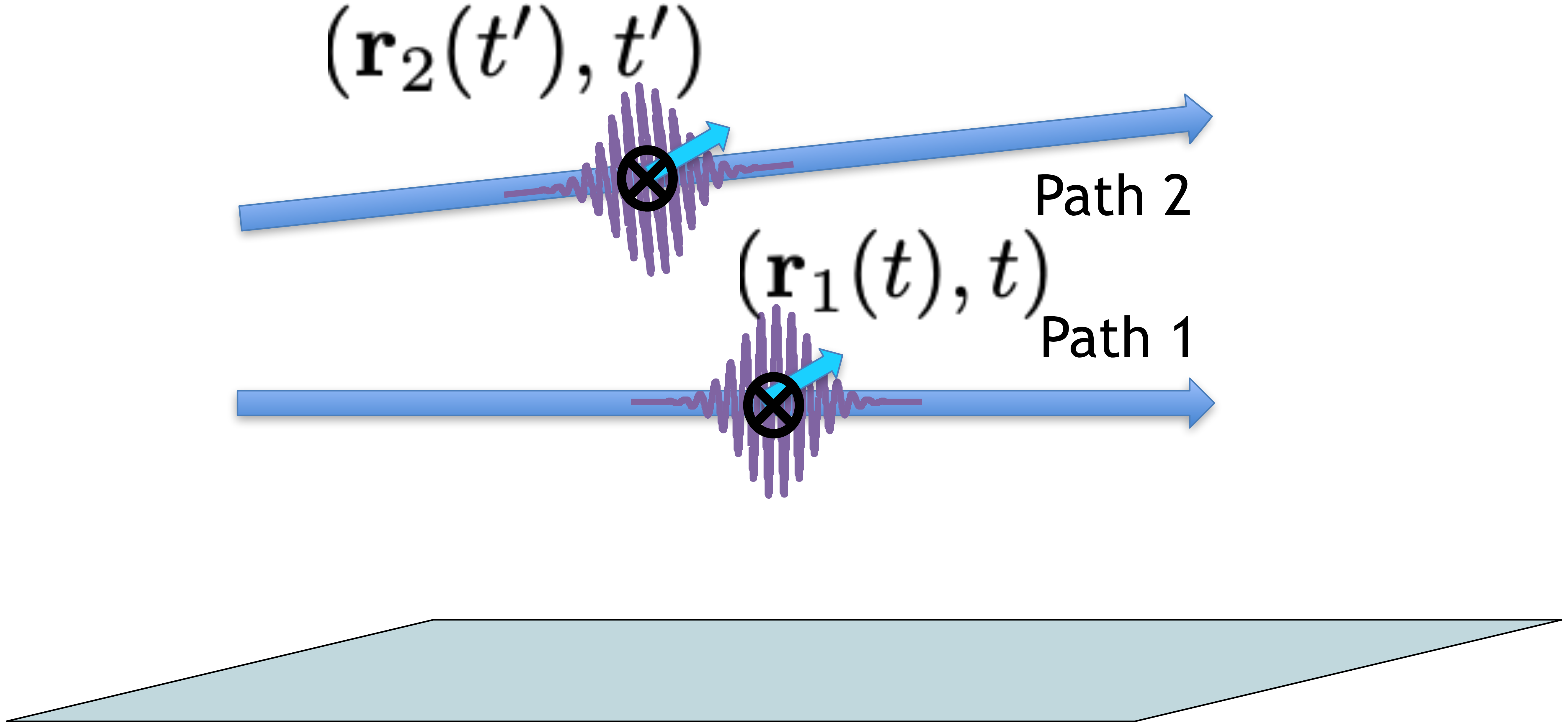
interferometer: self-interaction also with a different wave-packet component



F Impens, R Behunin, C Ccapa-Ttira and PAMN, EPL 2013

non-local Casimir atomic phase

- Atomic phases are normally *local*
- *Phase non-locality* emerges as a *dynamical-like Casimir effect*



non-local Casimir atomic phase

Casimir atomic phases beyond the quasi-static limit

Interaction Hamiltonian: $\hat{V}(\mathbf{r}(t)) = -\,\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\hat{\mathbf{r}}(t))$

(external) atomic position operator

Dipole operator

Electric field operator

Neutral atoms with no permanent dipole: $\langle\hat{\mathbf{d}}\rangle = \langle\hat{V}(\mathbf{r}(t),t)\rangle = 0$

Initial (produc) state:

$$|\Psi\rangle_{t=0} = \frac{1}{\sqrt{2}}\underbrace{\left(|\psi_1\rangle_0 + |\psi_2\rangle_0\right)}_{\text{external}} \otimes \underbrace{|\psi_A\rangle_0}_{\text{internal}} \otimes \underbrace{|\psi_F\rangle_0}_{\text{field}}$$

State at time *t*

Time-ordering operator

$$|\Psi\rangle_t = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle_t \otimes \underbrace{\boxed{\mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_1(t'),t')\right)|\psi_A\rangle_0\otimes|\psi_F\rangle_0}}_{|\psi^{(1)}_{AF}(t)\rangle} + |\psi_2\rangle_t \otimes \underbrace{\boxed{\mathcal{T}\exp\left(-\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_2(t'),t')\right)|\psi_A\rangle_0\otimes|\psi_F\rangle_0}}_{|\psi^{(2)}_{AF}(t)\rangle}\right)$$

non-local Casimir atomic phase

Reduced density operator for the external degree of freedom $\rho = \mathrm{Tr}_{AF}(|\Psi\rangle\langle\Psi|)$

Coherence multiplied by

$$e^{i\Delta\phi_{12}} = \langle\psi^{(2)}_{AF}(t)|\psi^{(1)}_{AF}(t)\rangle$$

$$e^{i\Delta\phi_{12}} = \langle\psi_{AF}(0)|\widetilde{\mathcal{T}}e^{\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_2(t'),t')}\mathcal{T}e^{-\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_1(t'),t')}|\psi_{AF}(0)\rangle$$

Anti time-ordering operator

Time-ordering operator

Complex phase $\Delta\phi_{12}$ has a positive imagine part (entaglement with environment/decoherence)

Real part of $\Delta\phi_{12}$ is the interferometric phase

non-local Casimir atomic phase

Reduced density operator for the external degree of freedom $\rho = \mathrm{Tr}_{AF}(|\Psi\rangle\langle\Psi|)$

Coherence multiplied by

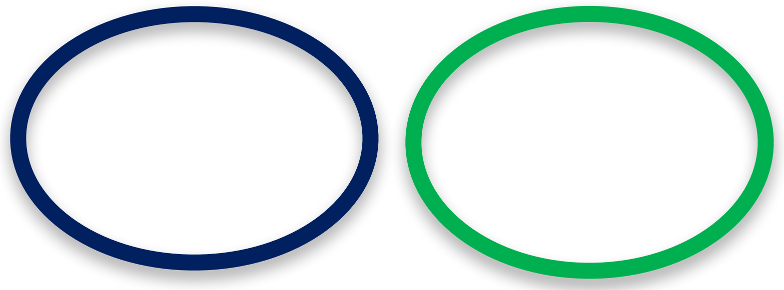
$$e^{i\Delta\phi_{12}} = \langle\psi_{AF}^{(2)}(t)|\psi_{AF}^{(1)}(t)\rangle$$

$$e^{i\Delta\phi_{12}} = \langle\psi_{AF}(0)|\widetilde{\mathcal{T}}e^{\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_2(t'),t')}\mathcal{T}e^{-\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_1(t'),t')}|\psi_{AF}(0)\rangle$$

Anti time-ordering operator

Time-ordering operator

Casimir phase obtained by picking up two interactions (2nd-order diagram)

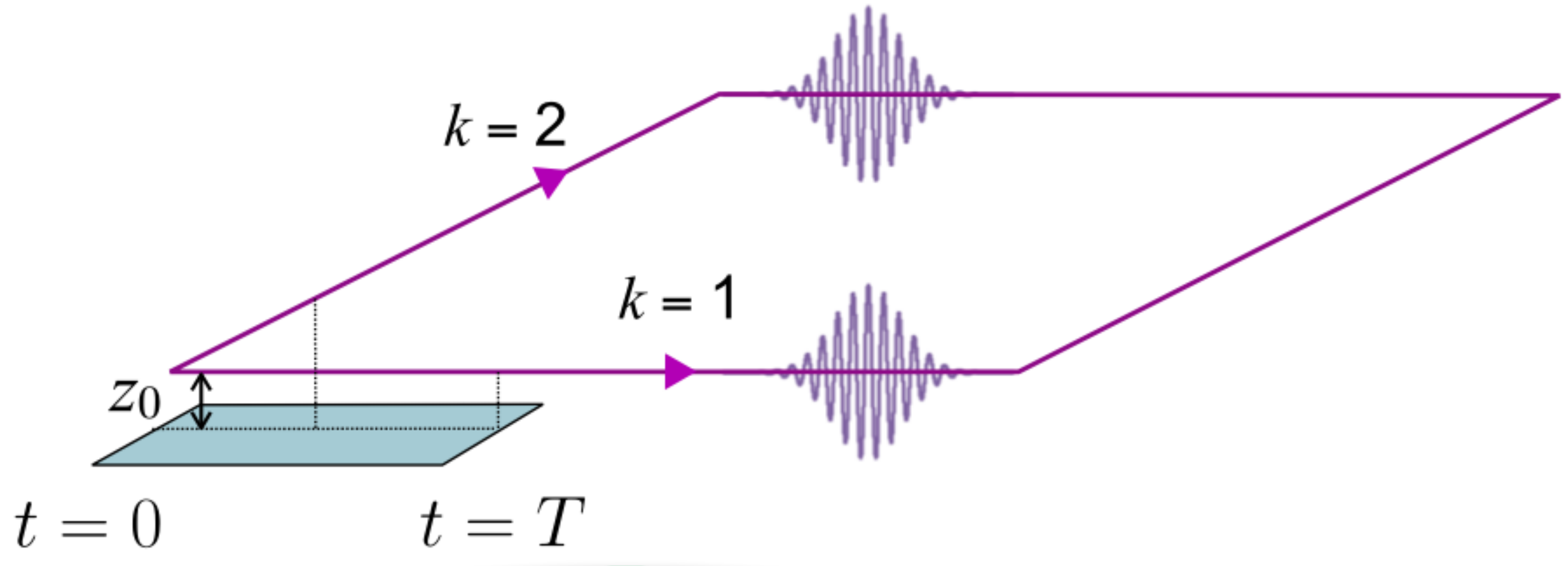


Two possibilities: Pick-up 2 interactions on the same path (->Local Casimir phases)

Pick up 2 interactions on two distinct paths (-> Nonlocal Casimir phases)

non-local Casimir atomic phase

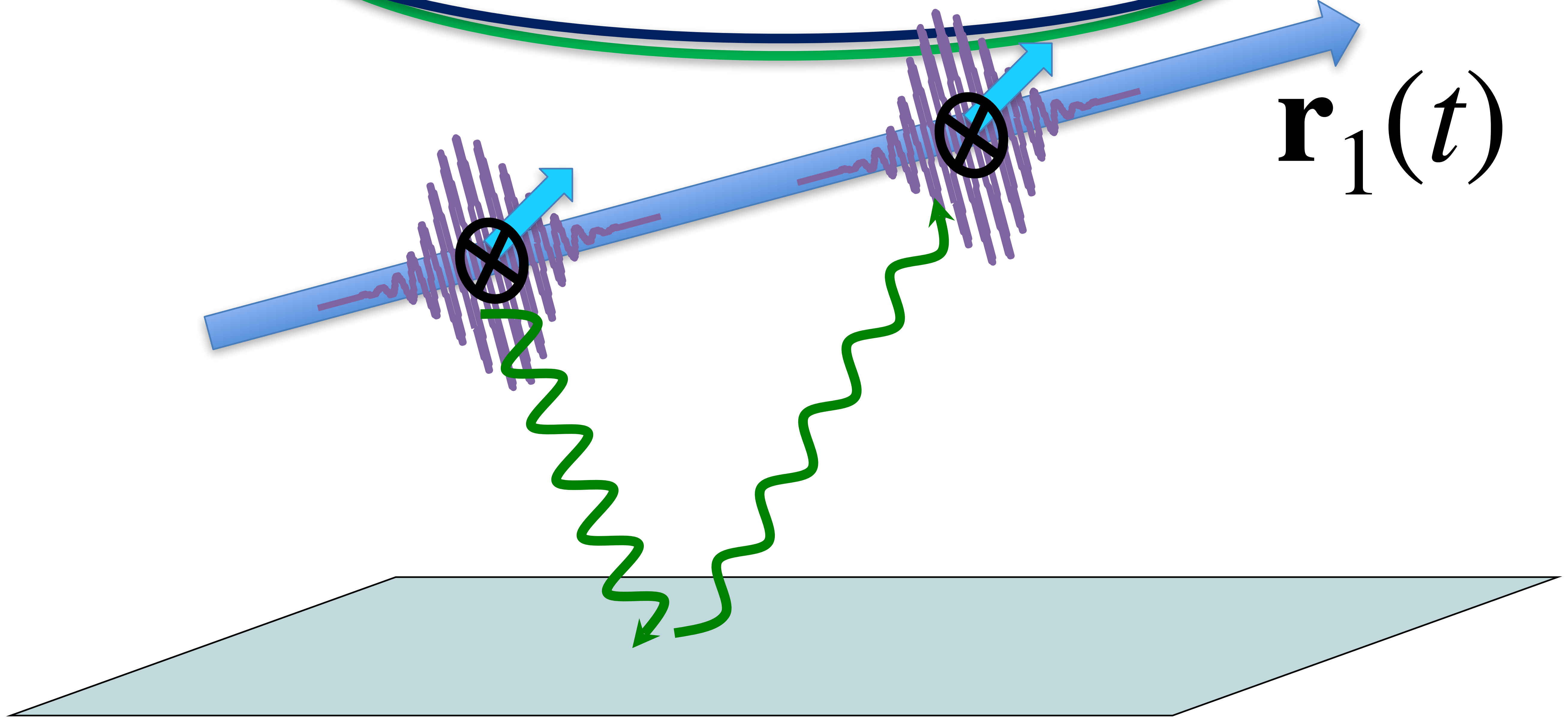
Local Casimir atomic phases



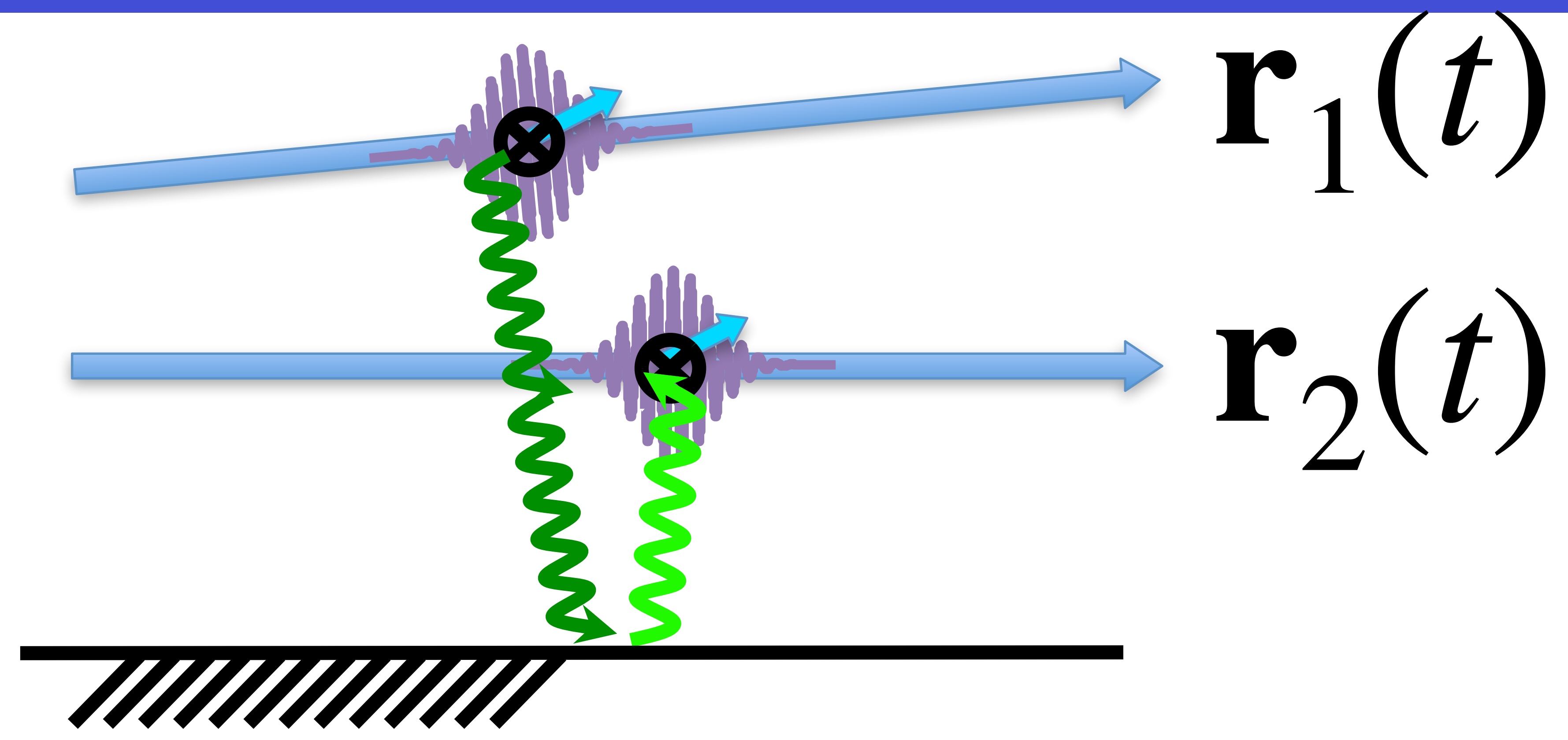
$$e^{i\Delta\phi_{12}} = \langle\psi_{AF}(0)|\tilde{\mathcal{T}} e^{\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_2(t'),t')} \mathcal{T} e^{-\frac{i}{\hbar}\int_0^t dt'\hat{V}(\mathbf{r}_1(t'),t')} |\psi_{AF}(0)\rangle$$

Local Casimir phases obtained by picking up two interactions on the same path

Contains the standard quasi-static phase reported in several experiments



non-local Casimir atomic phase

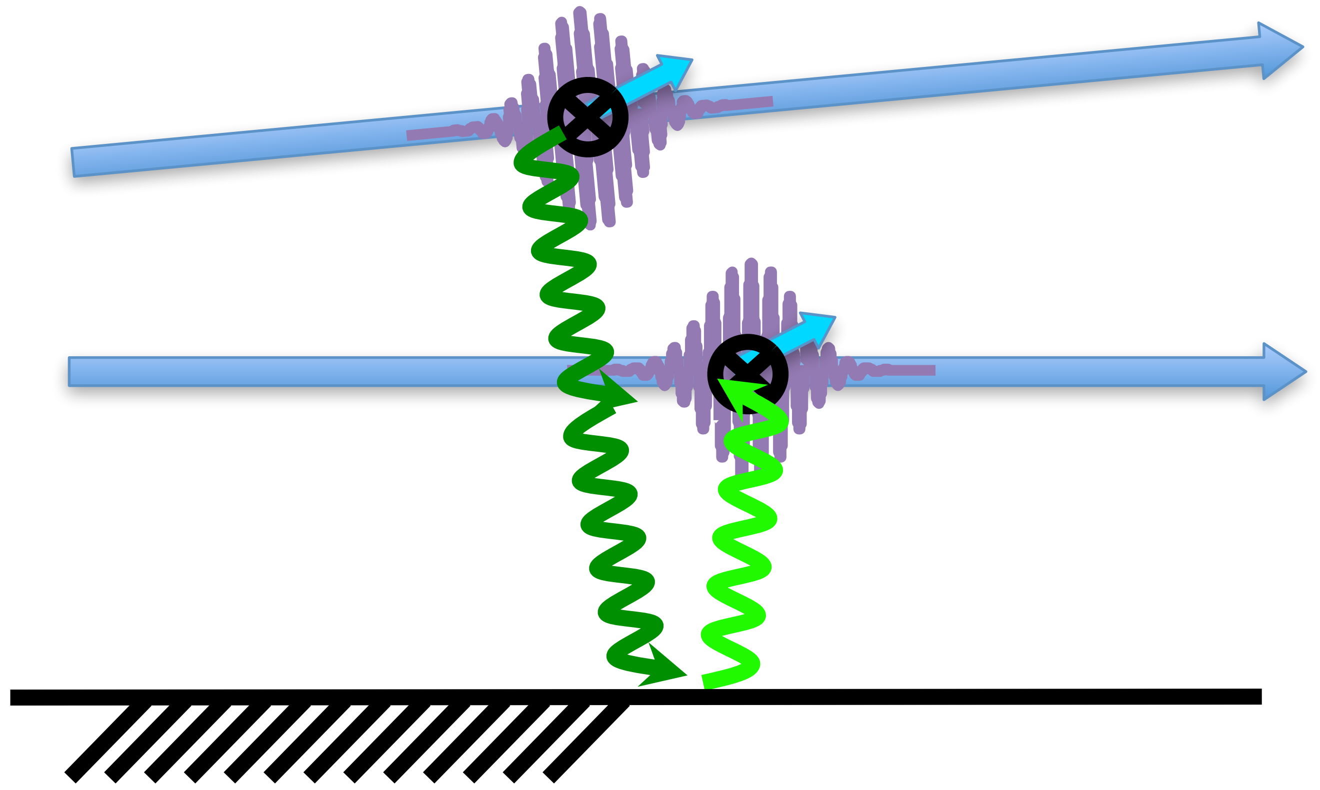


$$e^{i\Delta\phi_{12}} = \langle\psi_{AF}(0)|\widetilde{\mathcal{T}}e^{\frac{i}{\hbar}\int_0^t dt'\hat{V}_R(\mathbf{r}_2(t'),t')}\mathcal{T}e^{-\frac{i}{\hbar}\int_0^t dt'\hat{V}_R(\mathbf{r}_1(t'),t')}|\psi_{\mathrm{AF}}(0)$$

Nonlocal Casimir phases obtained by picking up two interactions on distinct paths

Vanishes in the quasi-static limit (but survives when accounting for the atomic motion) → Dynamical Casimir-like effect!

non-local Casimir atomic phase



$$\Delta\phi_{12} = \varphi_{11} - \varphi_{22} + \varphi_{12} - \varphi_{21}$$

Local phases

Nonlocal phases

$$\varphi_{kl} = \frac{1}{4}\iint_{-\frac{T}{2}}^{\frac{T}{2}} dt\, dt' \left[g^{H}_{\hat{\mathbf{d}}}(t,t')\mathcal{G}^{R,S}_{\hat{\mathbf{E}}}(\mathbf{r}_k(t),t;\mathbf{r}_l(t'),t') + g^{R}_{\hat{\mathbf{d}}}(t,t')\mathcal{G}^{H,S}_{\hat{\mathbf{E}}}(\mathbf{r}_k(t),t;\mathbf{r}_l(t'),t') \right]$$

Dipole fluctuations

Electric field fluctuations

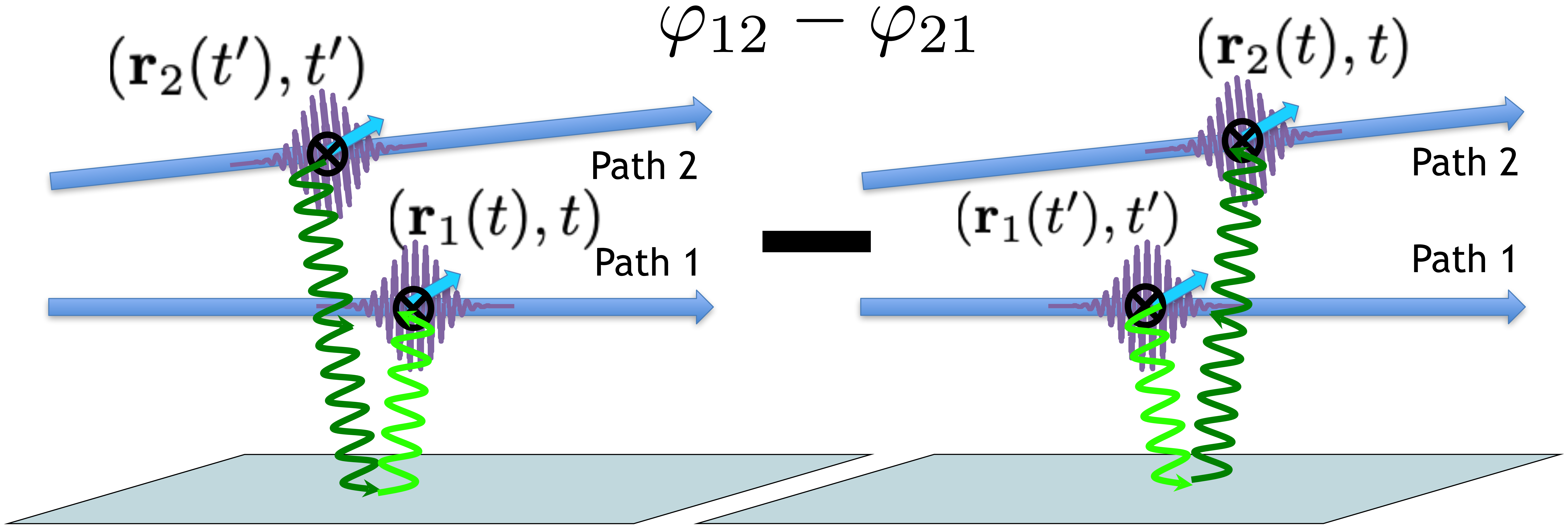
$$\boldsymbol{G}^{R}_{\hat{\mathbf{O}}\,ij}(t,t') = \frac{i}{\hbar}\Theta(t-t')\langle[\hat{O}_i(t),\hat{O}_j(t')]\rangle$$

Retarded Green’s functions= susceptibility functions

$$\boldsymbol{G}^{H}_{\hat{\mathbf{O}}\,ij}(t,t') = \frac{1}{\hbar}\langle\{\hat{O}_i(t),\hat{O}_j(t')\}\rangle$$

Hadamard Green’s functions= source of quantum fluctuations

non-local Casimir atomic phase



t' Retarded time

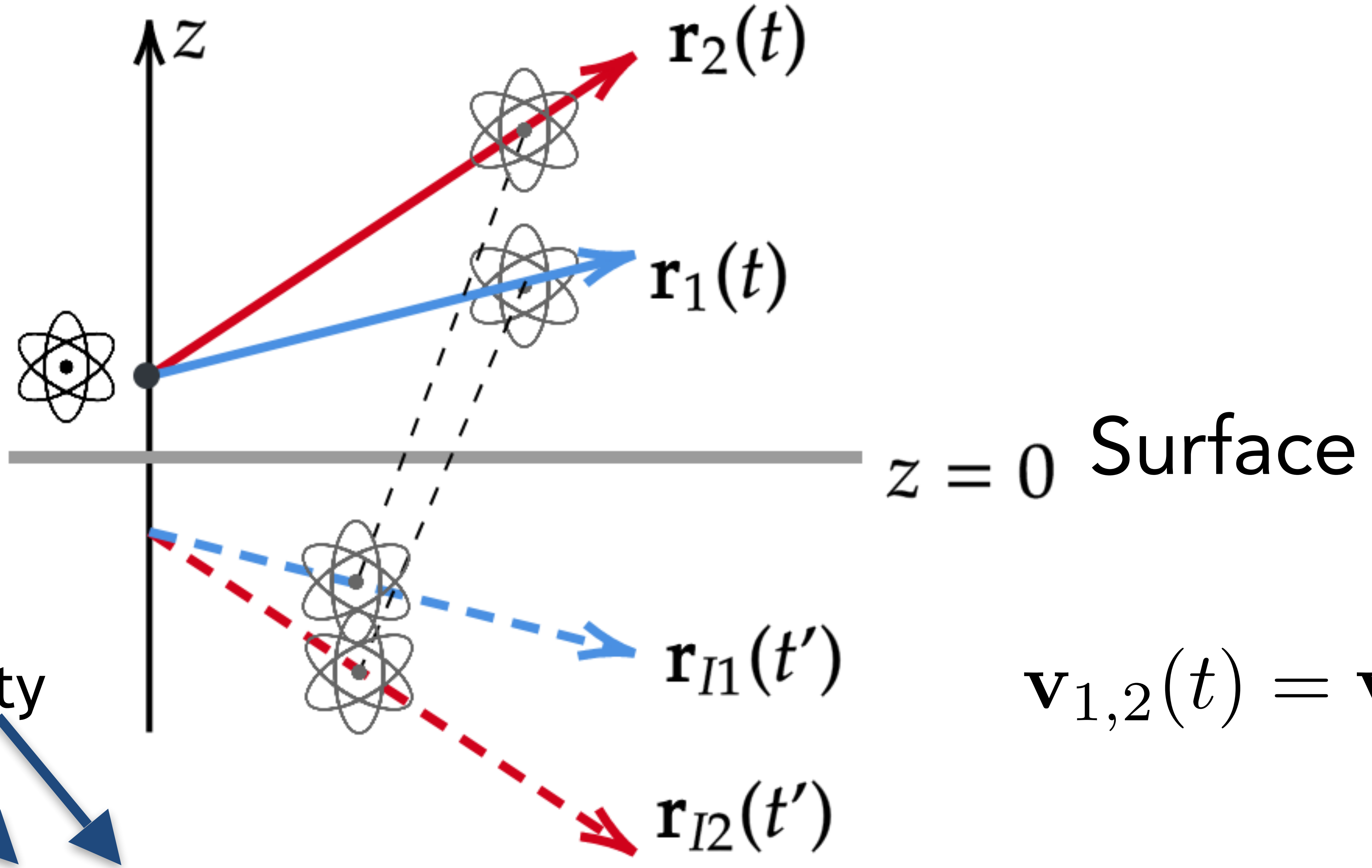
t Current time

$\tau = t - t'$ Duration of the virtual photon exchange

difference between diagrams arises from the motion normal to the surface

non-local Casimir atomic phase

Two diagrams with the "image method"



Single 2-level atom in a coherent superposition of two wave-packets

$$\mathbf{v}_{1,2}(t)=\mathbf{v}_{//}(t)+\dot{z}_{1,2}(t)\mathbf{u}_z$$

Atomic polarizability

Transition frequency

$$\phi_{12}=\frac{3\,\omega_0\alpha(0)}{4\pi\epsilon_0 c}\int_{-T/2}^{T/2}dt\,\frac{\dot{z}_1(t)-\dot{z}_2(t)}{(z_1(t)+z_2(t))^3}$$

Phase invariant under time rescaling $T\rightarrow\lambda T$

Changes sign with reversed propagation: $\mathbf{v}_{1,2}\rightarrow-\mathbf{v}_{1,2}\Rightarrow\phi_{12}\rightarrow-\phi_{12}$

Geometric phase!

FI, R. O. Behunin, Claudio Ccapa Ttira and Paulo A. Maia Neto, EPL, 101 60006 (2013); J. Phys B 46 245503 (2013);
For a review: FI, R. de Melo e Souza, G. C. Matos, EPL (2022).

Outline

- Microscopic Dynamical Casimir Effect
- Geometric and non-local Casimir atomic phases
- Quantum Sagnac Effect

GHz rotation of optically trapped nanoparticles

nature nanotechnology

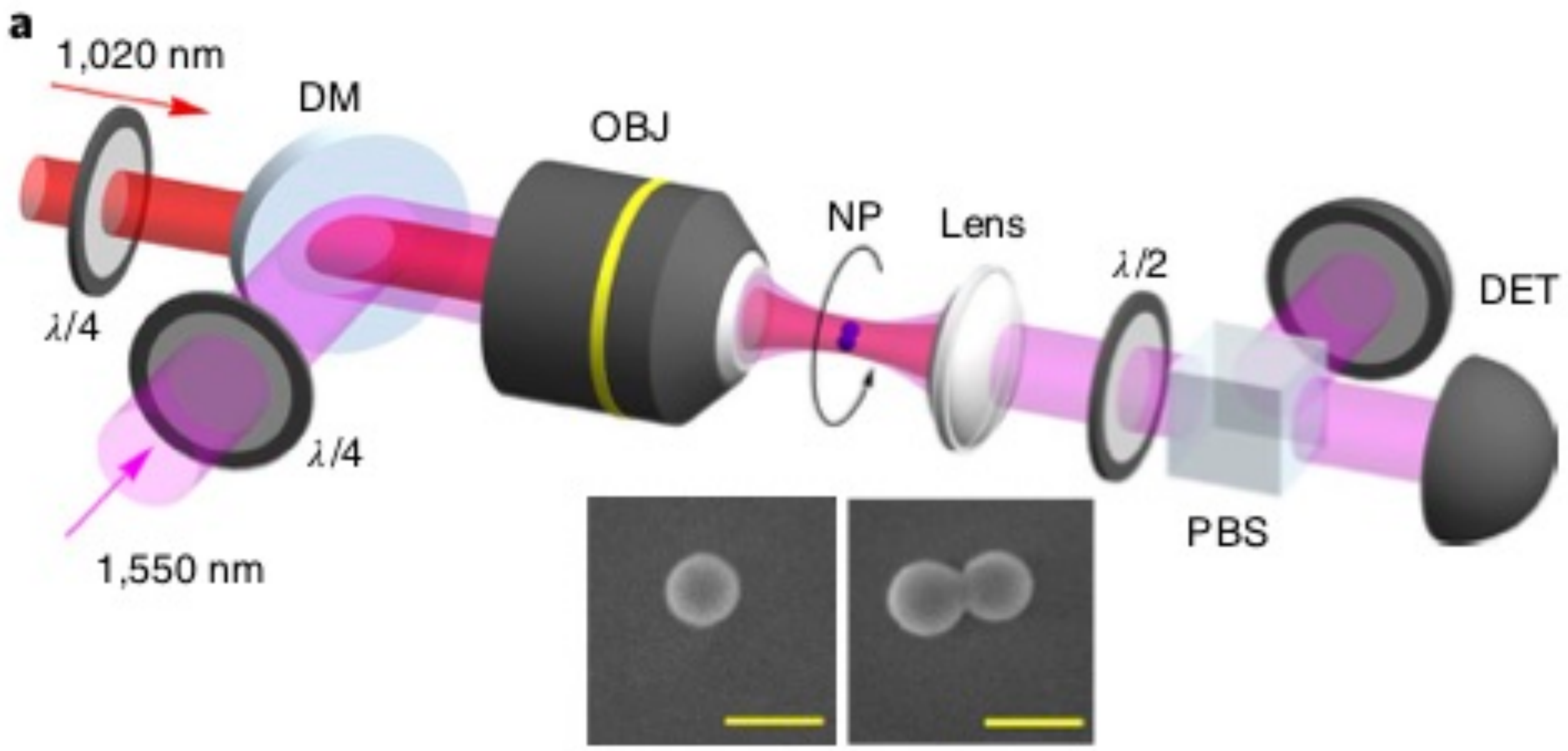
LETTERS

https://doi.org/10.1038/s41565-019-0605-9

Ultrasensitive torque detection with an optically levitated nanorotor

Jonghoon Ahn[1], Zhujing Xu[2], Jaehoon Bang[1], Peng Ju[2], Xingyu Gao[2] and Tongcang Li[1,2,3,4*]

vacuum. Our system does not require complex nanofabrication. Moreover, we drive a nanoparticle to rotate at a record high speed beyond 5 GHz (300 billion r.p.m.). Our calculations



Featured in Physics

GHz Rotation of an Optically Trapped Nanoparticle in Vacuum

René Reimann, Michael Doderer, Erik Hebestreit, Rozenn Diehl, Martin Frimmer, Dominik Windey, Felix Tebbenjohanns, and Lukas Novotny

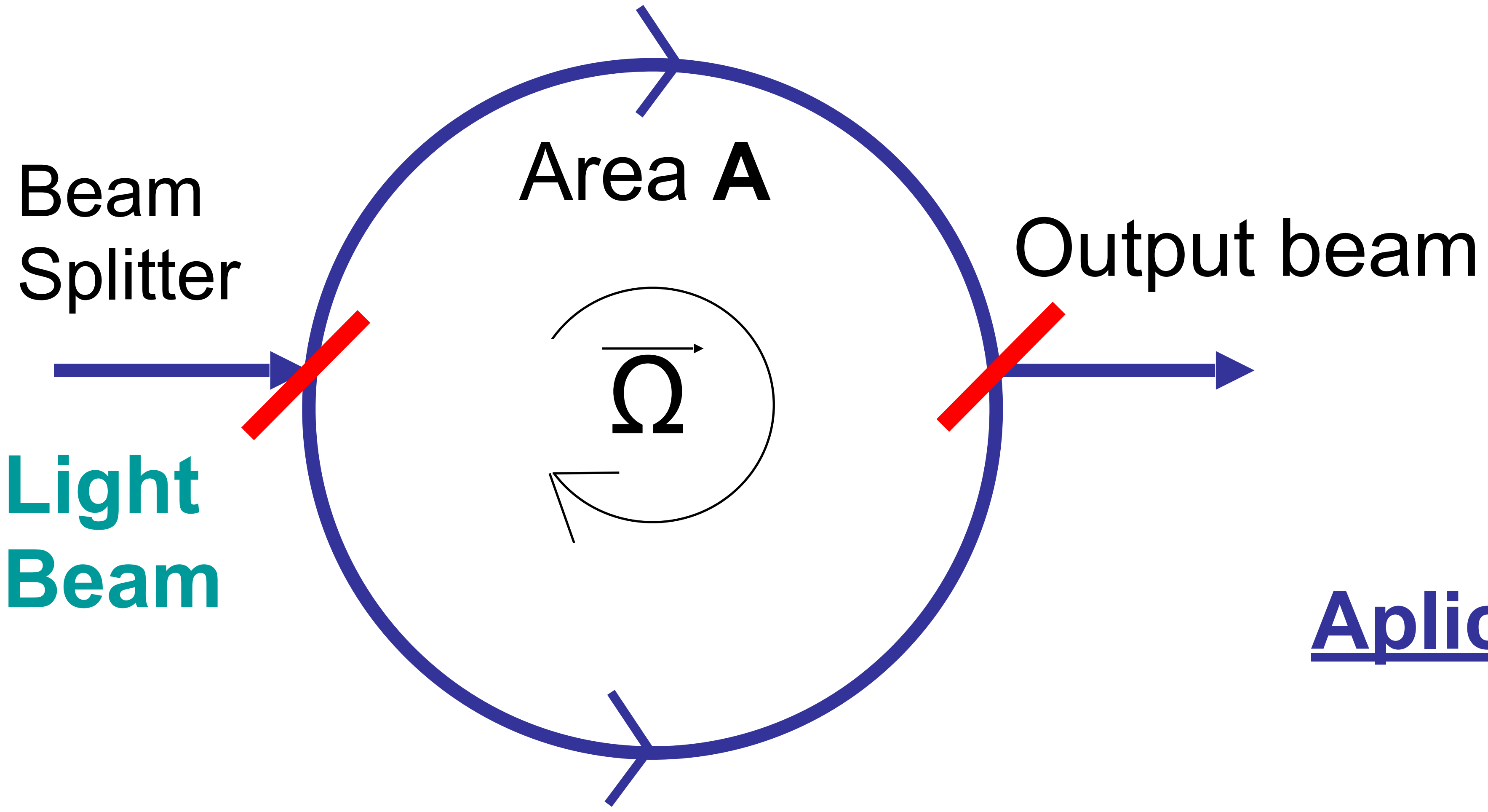
Phys. Rev. Lett. **121**, 033602 – Published 20 July 2018; Erratum Phys. Rev. Lett. **126**, 159901 (2021)

Physics See Focus story: The Fastest Spinners

Opportunity to probe dynamical Casimir effects….?

Sagnac Effect with Light/Atomic Waves

Sagnac effect (1913):



Unified expression for Sagnac Phase for atomic/light waves:

$$\Delta\phi = \frac{4\pi}{\lambda v}\mathbf{\Omega}\cdot\mathbf{A}$$

Aplications: Inertial navigation systems in aircrafts

Georges Sagnac
(Fonte:Alchetron)

Phase difference between the two interferometers arms proportional to the angular rotation frequency Ω and to the enclosed area

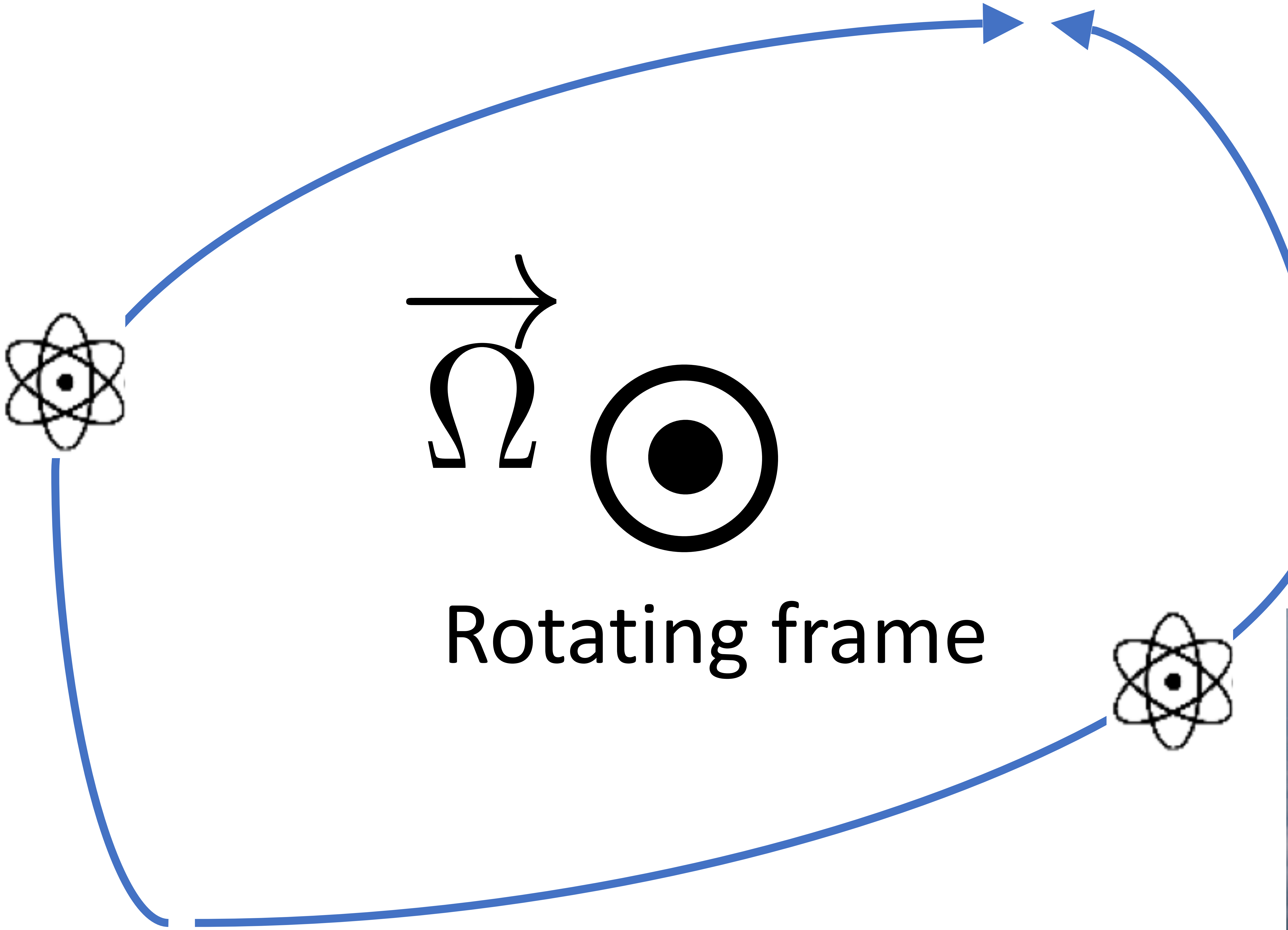
Sagnac Effect for atomic waves:

(Ch. Bordé 1989, Bouyer&Kasevich 1998) (^{87}Rb)

$$\frac{\Delta\phi_{at}}{\Delta\phi_l} = \frac{\lambda_l v_l}{\lambda_{at} v_{at}} = \frac{mc^2}{\hbar\omega} \sim 10^{11}$$

Stronger non-inertial effect for atomic waves!

Sagnac Atom Interferometer

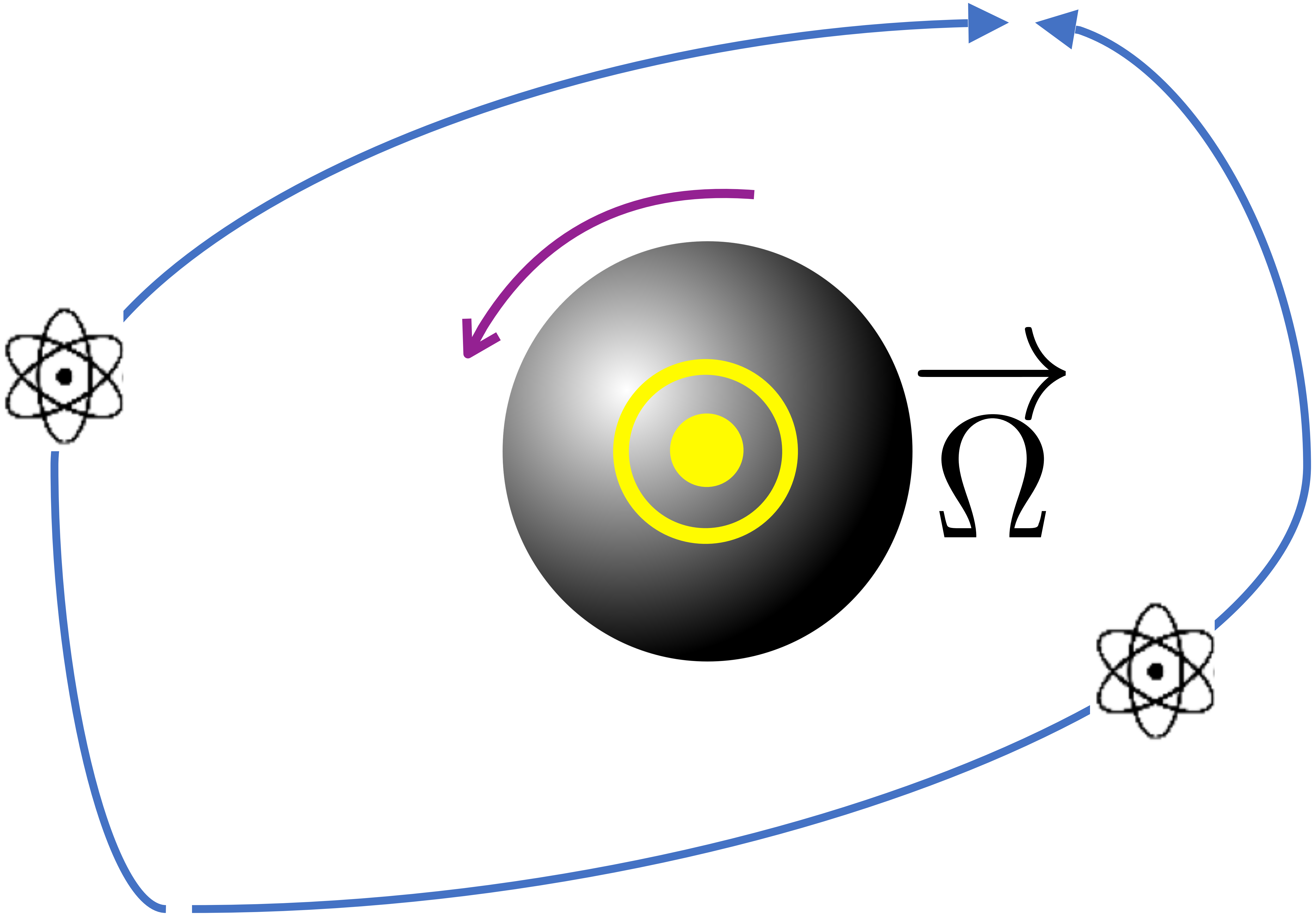


$\vec{\Omega}$ ⊙

Rotating frame

Ex: embarked atom interferometer

Sagnac effect in an inertial frame?

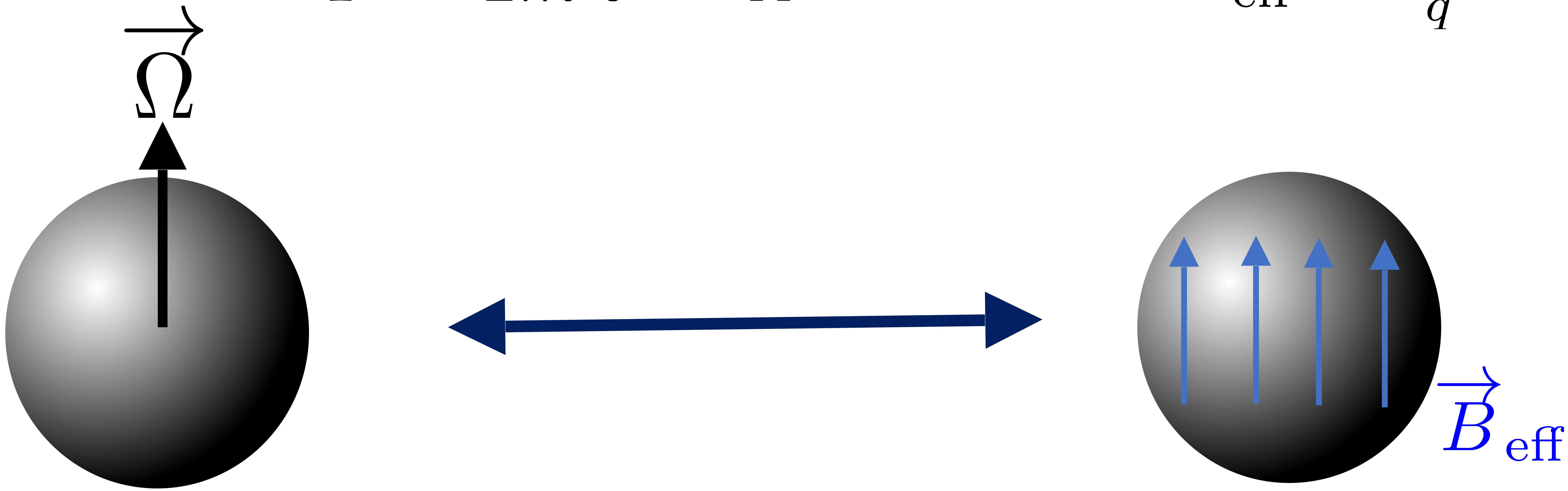


Inertial frame and rotating conductor

An alternative point-of-view: an Aharonov-Bohm-like effect

Lorentz Force: $\overrightarrow{F} = q\overrightarrow{v} \times \overrightarrow{B}$

Coriolis Force: $\overrightarrow{F} = 2m\overrightarrow{v} \times \overrightarrow{\Omega} \quad \longleftrightarrow \quad \overrightarrow{B}_{\text{eff}} = \frac{2m}{q}\overrightarrow{\Omega}$

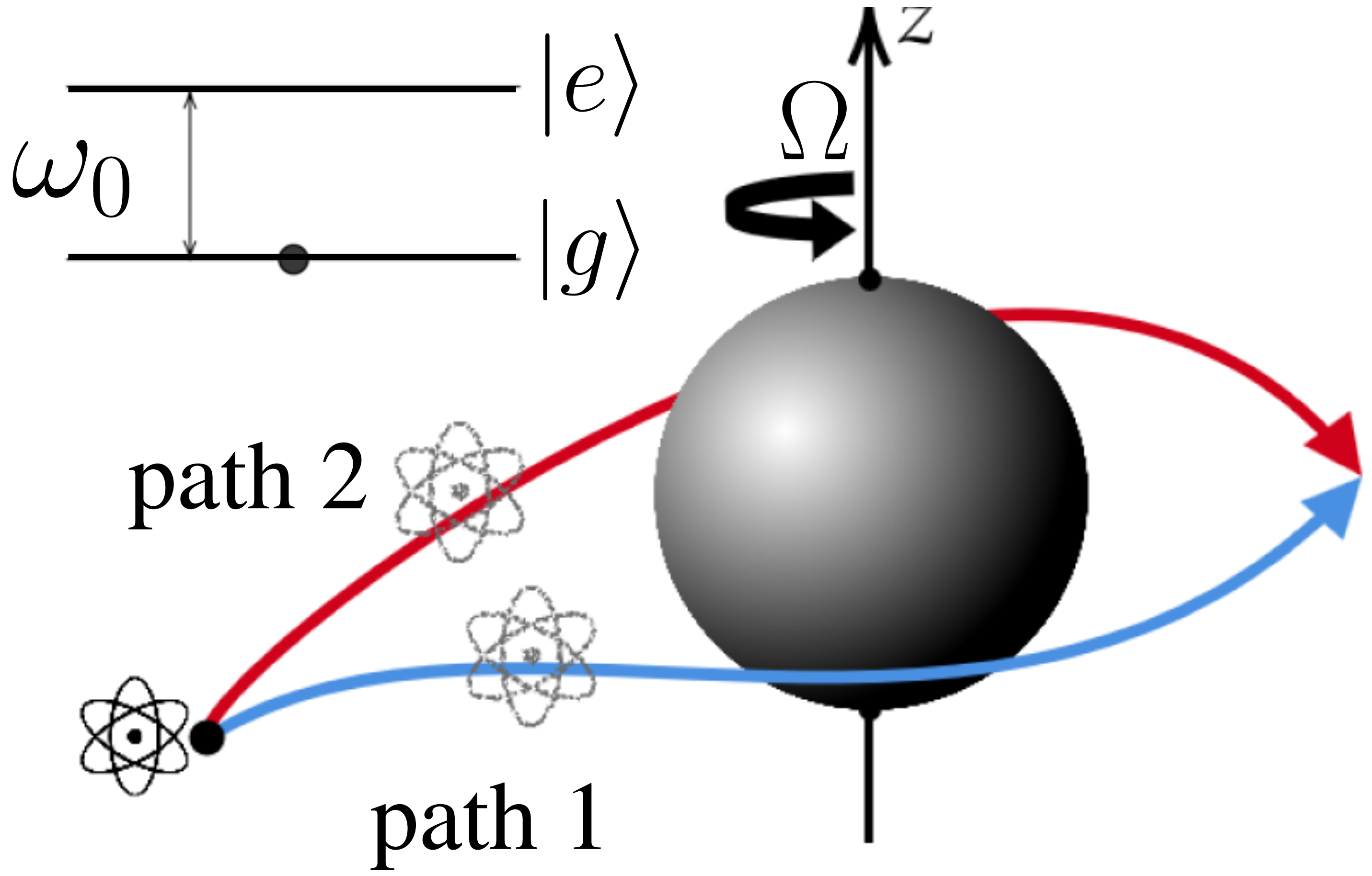


Rotation of a body
in an inertial frame

Effective magnetic field
confined to the body

Trace of the rotation??

Quantum Sagnac phase near a spinning particle



Casimir phase:

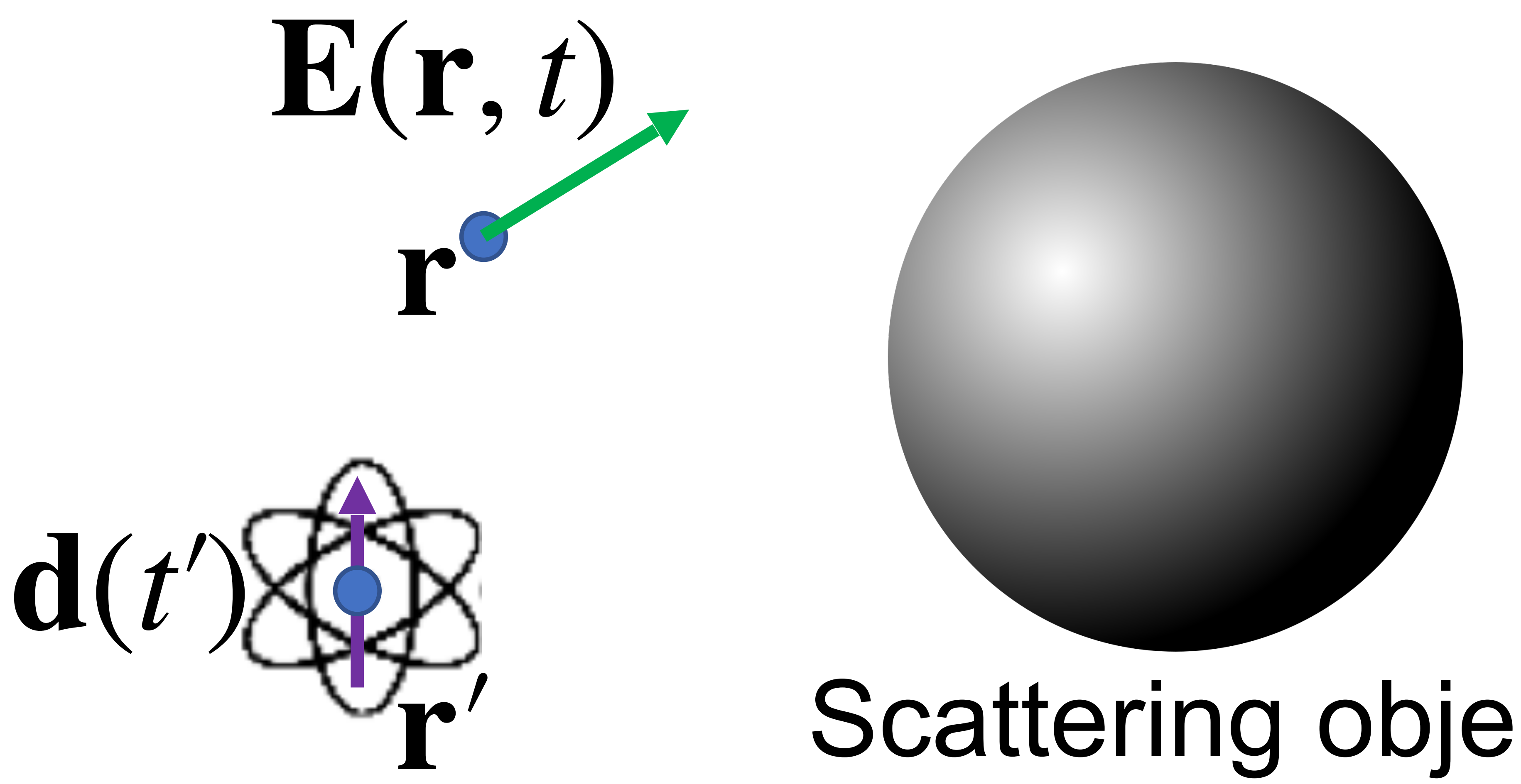
$$\Delta\phi_{12} = \varphi_{11} - \varphi_{22} + \varphi_{12} - \varphi_{21}$$

Spinning nano-particle

$$\varphi_{kl} = \frac{1}{4}\iint_{-\frac{T}{2}}^{\frac{T}{2}} dt\, dt' \left[g^{H}_{\hat{\mathbf{d}}}(t,t')\mathcal{G}^{R,S}_{\hat{\mathbf{E}}}(\mathbf{r}_k(t),t;\mathbf{r}_l(t'),t') + (R \leftrightarrow H) \right]$$

What are the electric-field Green's function in presence of a spinning body?

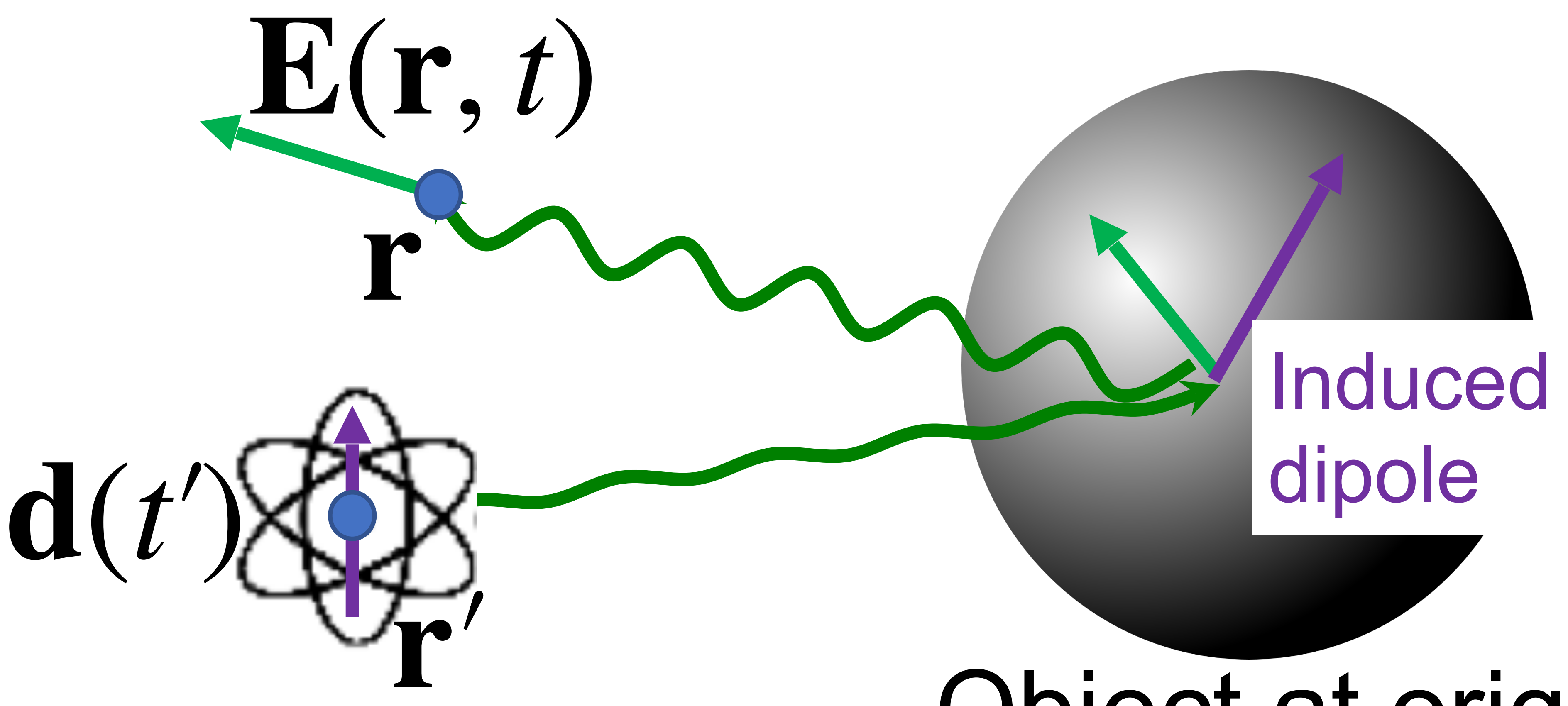
Scattered electric field Green's functions



Scattering object

Retarded field Green's function
= Response to the dipole excitation

$$E_i(\mathbf{r},t) = G^R_{\hat{\mathbf{E}},\, ij}(\mathbf{r},t;\mathbf{r}',t')d_j(t')$$



Instantaneous
dipole excitation

Object at origin
$(\mathbf{r} = \mathbf{0})$

Dipole approximation

Scattered field Green's function:

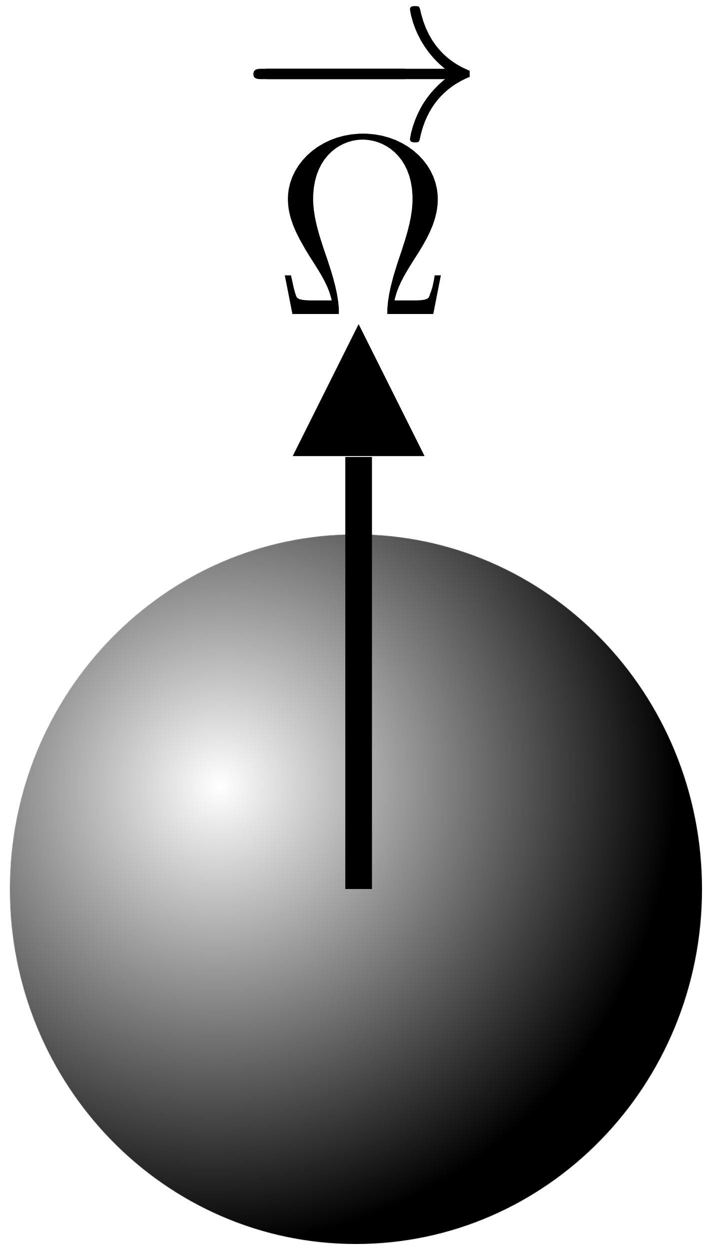
Object
Polarizability tensor

$$\boldsymbol{G}^{R,S}_{\hat{\mathbf{E}}}(\mathbf{r},\mathbf{r}',\omega) = \boldsymbol{G}^0(\mathbf{r},\mathbf{0},\omega)\cdot\boldsymbol{\alpha}(\omega)\cdot\boldsymbol{G}^0(\mathbf{0},\mathbf{r}',\omega)$$

Free electric field
Green functions

Polarizability tensor of a spinning nano-particle?

A. Manjavacas e F. J. García de Abajo, Phys Rev. A **82**,063827 (2010).



Rotating spherical
nanosphere in the
dipole approximation

Dipole response obtained in the sphere frame.

Switch from sphere frame / inertial frame

Leading non-relativistic order

Polarizability induced by the rotation:

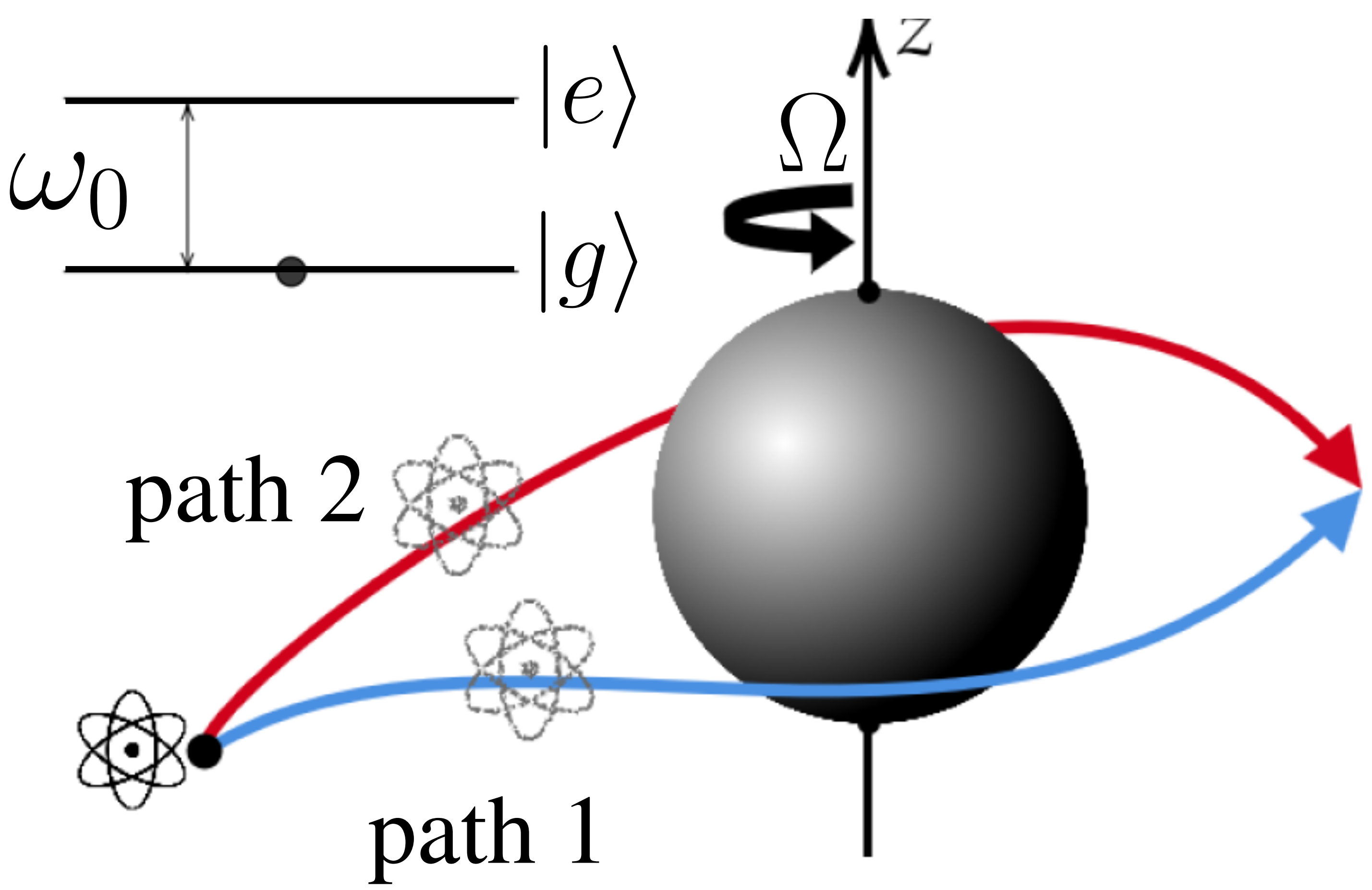
$$\alpha^{\mathbf{\Omega}}_{ij}(\omega) = i\alpha'_S(\omega)\epsilon_{ijk}\Omega_k$$

Antisymmetric
Levi-Civitta tensor

$\alpha_S(\omega)$ = Polarizability of the sphere at rest

Requires dispersion!

Quantum Sagnac phase



G. C. Matos, Reinaldo de Melo e Souza,
PAMN, and F Impens,
Phys. Rev. Lett. **127**, 270401 (2021).

Local Sagnac phase:

$$\Delta\phi_{12}^{\Omega} = \varphi_{11}^{\Omega} - \varphi_{22}^{\Omega} + \varphi_{12}^{\Omega} - \varphi_{21}^{\Omega}$$

ϕ_1^{Ω} ϕ_2^{Ω}

Local Quantum Sagnac phase in the limit $c \to +\infty$

$$\phi_{\mathrm{vdW,k}}^{\Omega} = \frac{9}{2}\,\frac{\omega_0\alpha_0^{\mathrm{A}}\tilde{\alpha}_{S,R}''(\omega_0)}{(4\pi\epsilon_0)^2}\int_{\mathcal{P}_k} d\mathbf{r}\cdot\frac{\mathbf{\Omega}\times\mathbf{r}}{r^8}$$

Real part of the spherical particle polarizability

$$\tilde{\alpha}_{S,R}(\omega) = \mathrm{Re}[\alpha_S(\omega)]$$

α_0^A = static atomic polarizability

Enhancement of the Quantum Sagnac phase with plasmon resonance

Goal: Choose atom/nano-particle to maximize second polarizability derivative $\tilde{\alpha}''_{S,R}(\omega)$ at the 2-level atom frequency ω_0

Published: 21 March 2012

Quantum plasmon resonances of individual met nanoparticles

Jonathan A. Scholl, Ai Leen Koh & Jennifer A. Dionne

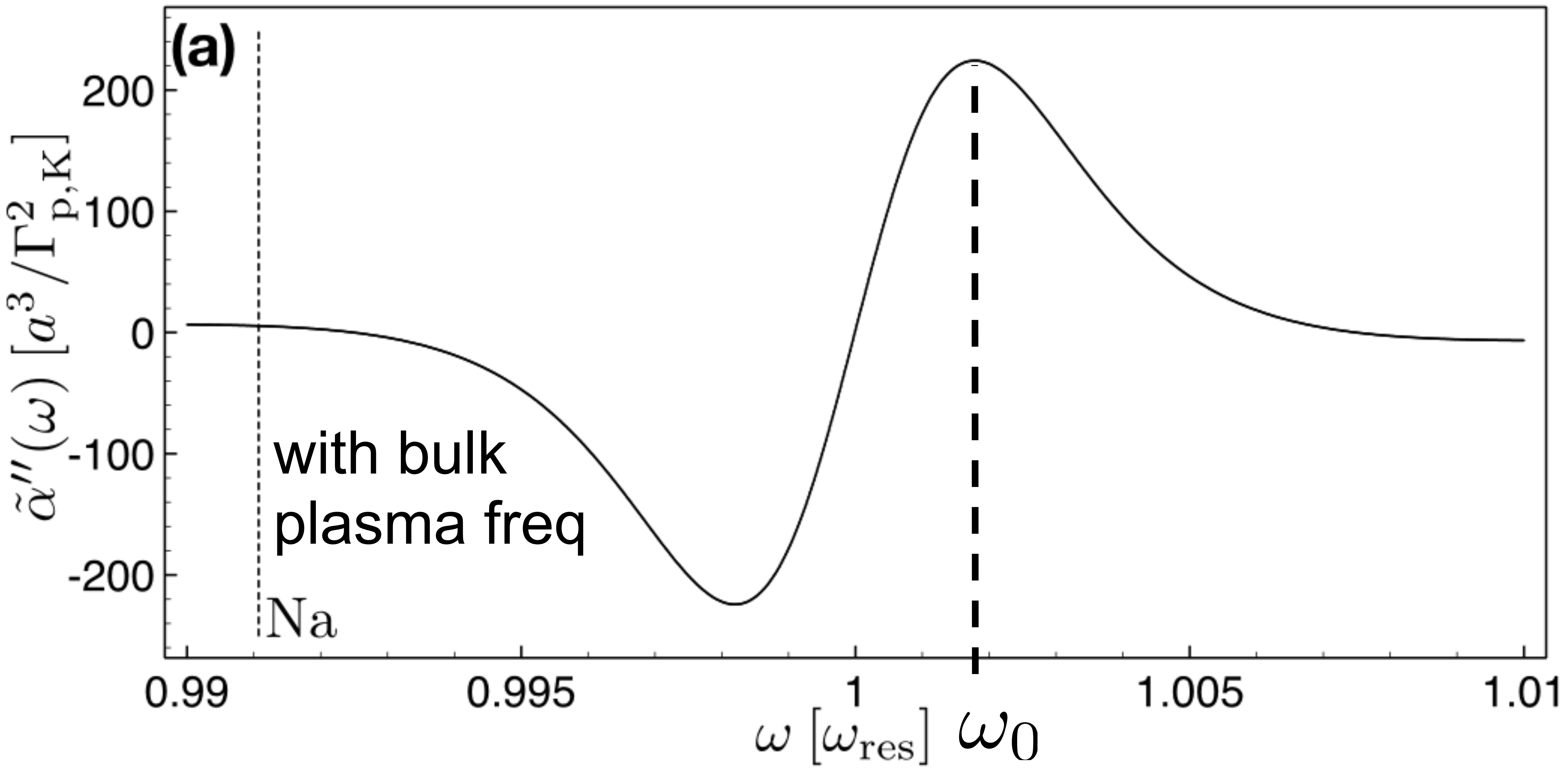
Nature **483**, 421–427 (2012) | Cite this article

Enhancement with plasmon resonance

$$\tilde{\alpha}(\omega) = (4\pi\epsilon_0)a^3\frac{\epsilon(\omega)-1}{\epsilon(\omega)+2}$$

Plasmon resonance at the frequency

$$\epsilon(\omega_{\mathrm{res}}) = -2$$



Considered example for numerical applications:

Na atom $(3s_{1/2} - 3p_{3/2})$/ K nano-sphere

$\omega_0 = 3.198 \times 10^{15}\ \mathrm{rad/s}$

Estimation of the Quantum Sagnac phase in an atom-Interferometer

Atomic wave-packets of finite width

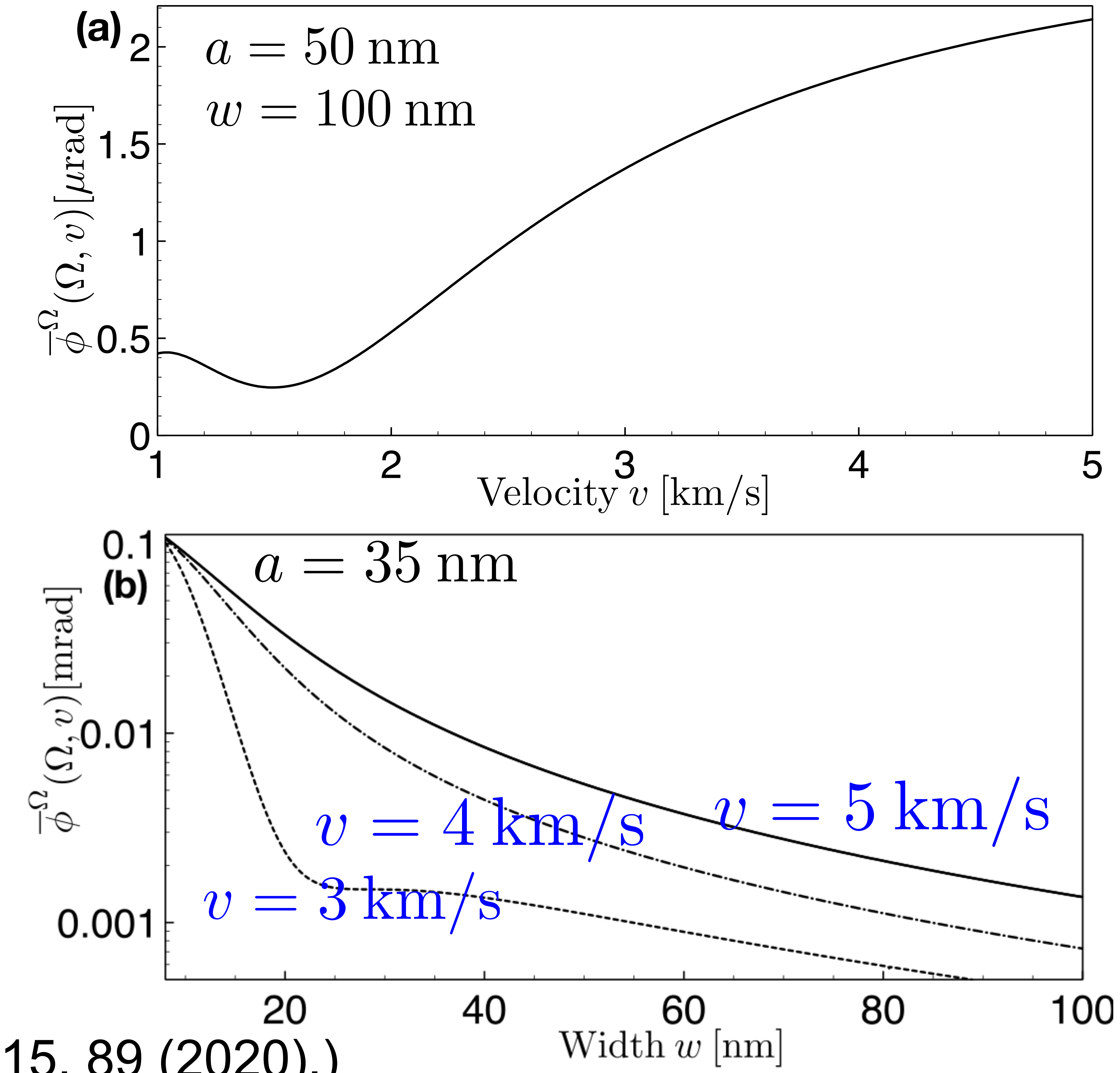
Total phase = quasi-static van der Waals
+ quantum Sagnac phase

$$\phi(\Omega, x, z, v) = \phi^{\mathrm{vdW}}(x, z, v) + \phi^{\Omega}(x, z)$$

Accessible quantum Sagnac phase

$$\overline{\phi}^{\Omega}(\Omega, v) \equiv \overline{\phi}(\Omega, v) - \overline{\phi}(0, v)$$

averaging over wave-packet width
(as in Alexander D. Cronin and John D. Perreault, Phys. Rev. A 70, 043607 (2004))



Considered parameters:

$\Omega = 2\pi \times 5\ \mathrm{GHz}$ (obtained in J. Ahn et al., Nat. Nanotechnol. 15, 89 (2020).)

Nanosphere radius $a = 30 - 50\ \mathrm{nm}$

Atomic beam of width $w = 10 - 100\ \mathrm{nm}$

Atomic velocities $v = 1 - 5\ \mathrm{km/s}$

Na atoms

K nanoparticle

$$\overline{\phi}^{\Omega}(\Omega, v) \simeq 0.1\ \mathrm{mrad}$$

Funding:

- FAPERJ
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- CNPq, CAPES: Cofecub (France)

Thank you!