Nonequilibrium dynamics in critical long-range interacting systems

Ricardo Puebla

Universidad Carlos III de Madrid (Spain)

III Quantum Non-Stationary Systems

August 29, 2024

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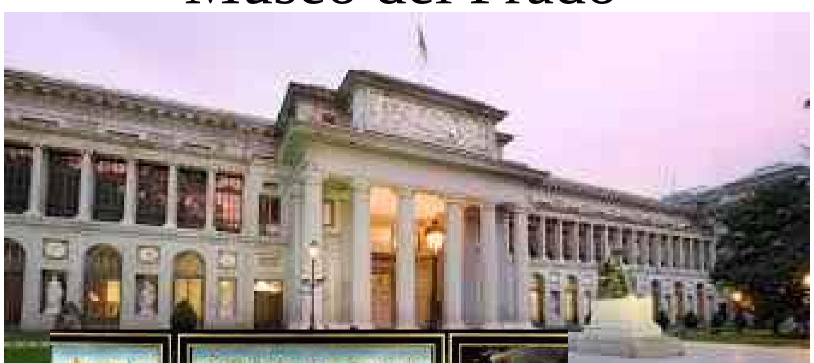
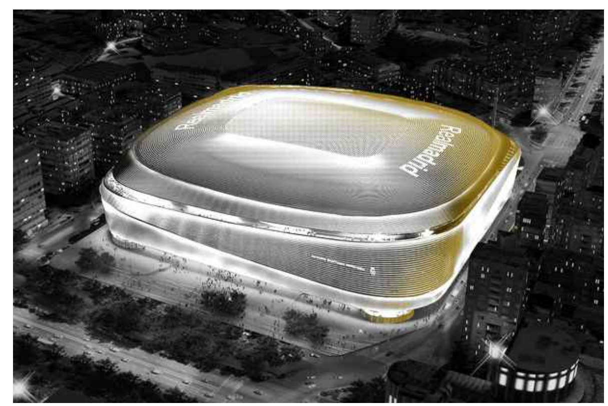




Nonequilibrium dynamics in critical long-range interacting systems

Louis Garbe
TUM (Germany)

Obinna Abah
Univ. Newcastle (UK)

Simone Felicetti
CNR (Italy)

Gabriele De Chiara
QUB (UK)

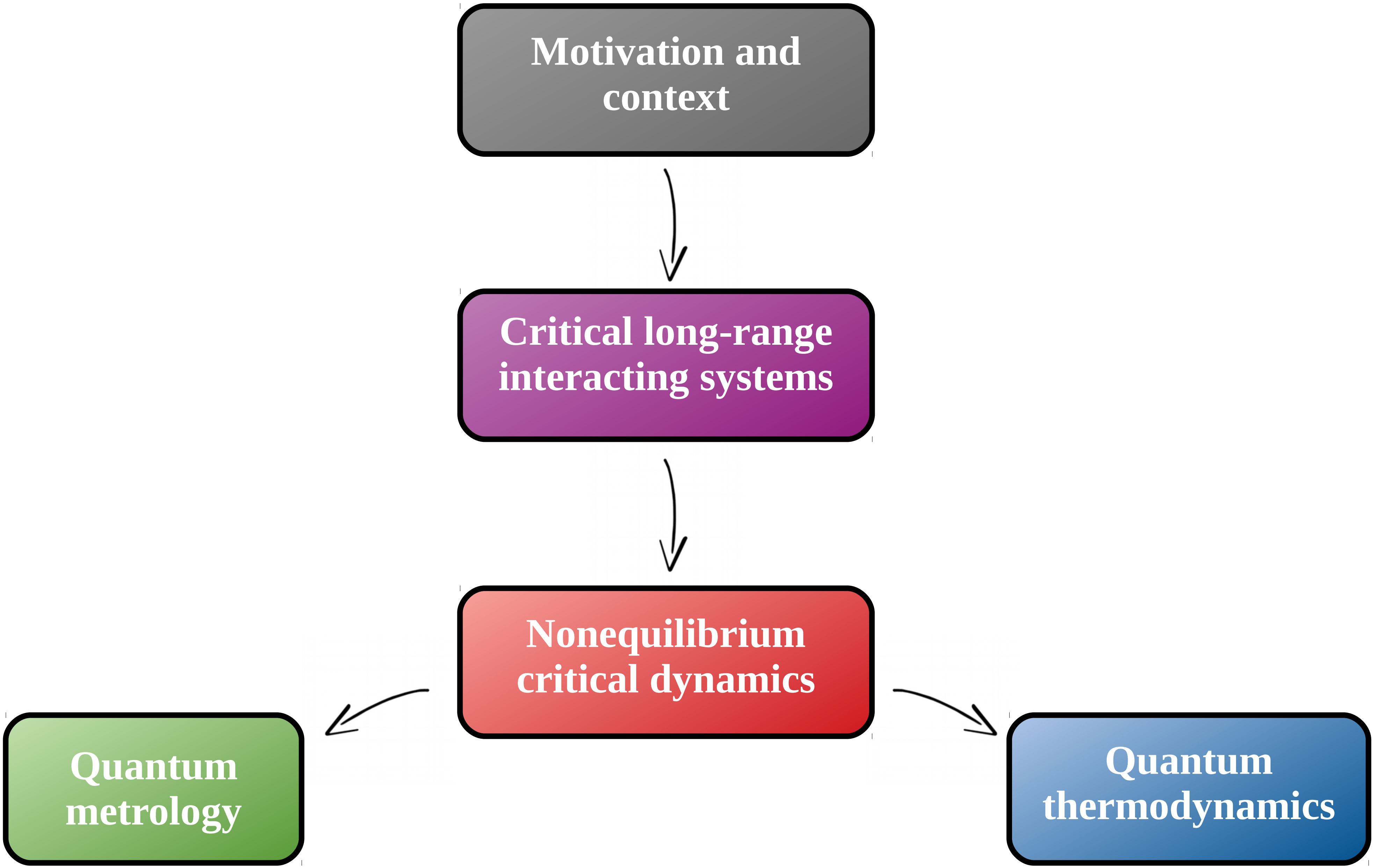
Mauro Paternostro
Univ. Palermo (Italy)

Fernando J. Gomez-Ruiz
Univ. Valladolid (Spain)

Stefano Gherardini
CNR & LENS (Italy)

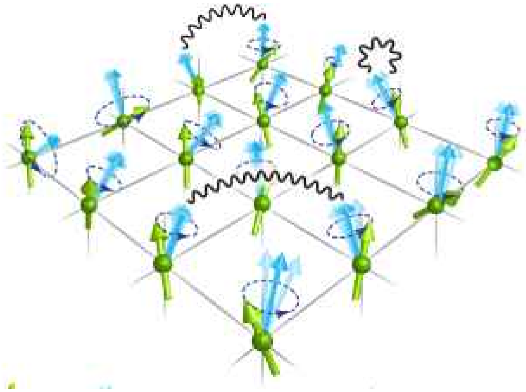
O. Abah, G. De Chiara, M. Paternostro, RP, *Phys. Rev. Research 4 (2), L022017 (2022)*
L. Garbe, O. Abah, S. Felicetti, RP, *Quantum Sci. Technol. 7, 035010 (2022)*
L. Garbe, O. Abah, S. Felicetti, RP, *Phys. Rev. Research 4, 043061 (2022)*
F. J. Gomez-Ruiz, S. Gherardini, RP (*in preparation)*

Outline

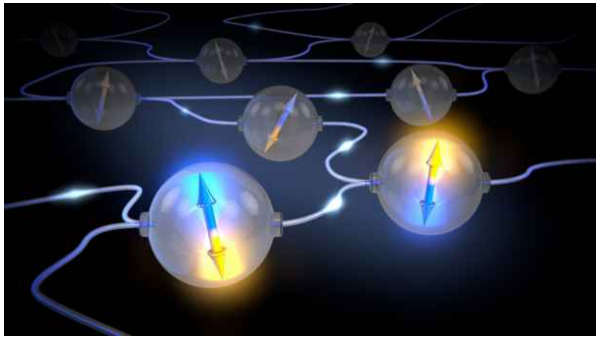

Motivation and context
Critical long-range interacting systems
Nonequilibrium critical dynamics
Quantum metrology
Quantum thermodynamics

Motivation

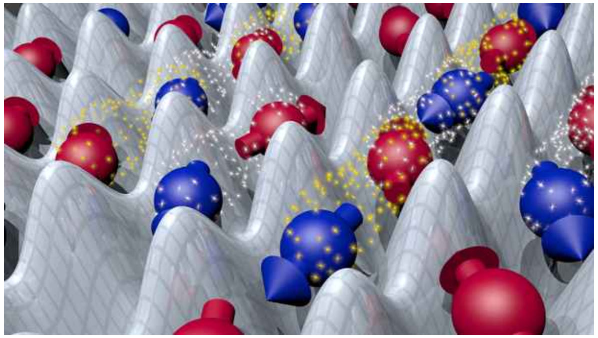
- Quantum many-body systems play a key role in modern quantum-based technologies



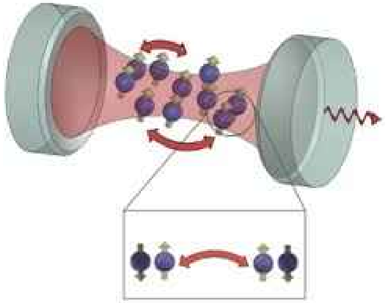
Condensed matter



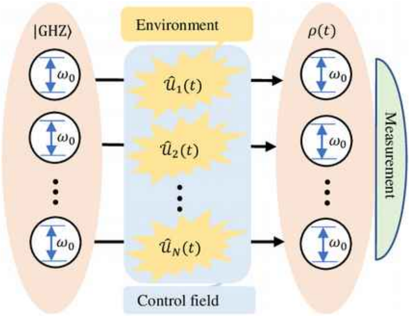
Quantum information and computation



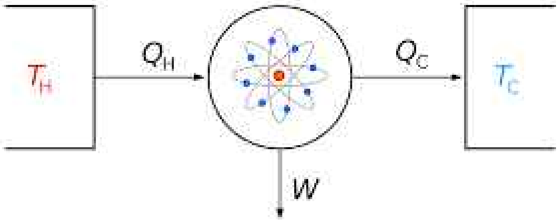
Quantum simulation



AMO physics



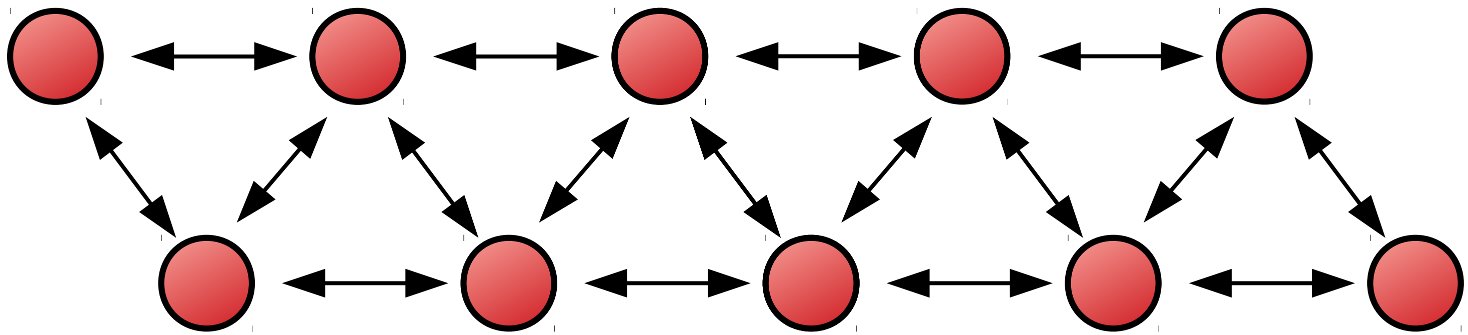
Quantum metrology



Quantum thermodynamics

Motivation

- Quantum many-body systems play a key role in modern quantum-based technologies



- The competition between interactions and local terms make them challenging (~2^N)

$$H(g) = H_0 + gH_1$$

External controllable parameter

Motivation

- Quantum phase transitions may occur in the ground state of the Hamiltonian $H(g)$

$$H(g) = H_0 + gH_1 \qquad [H_0, H_1] \neq 0$$

quantum fluctuations trigger the phase transition!

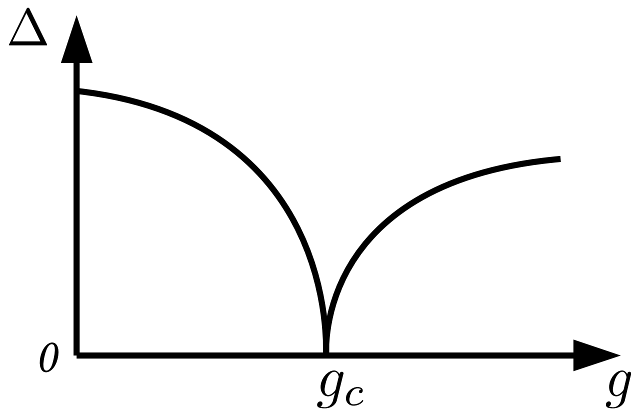
- Long-range correlations at a critical point

$$G(\mathbf{r}_1, \mathbf{r}_2) \sim e^{-|\mathbf{r}_1 - \mathbf{r}_2|/\xi}$$

$$\xi = \xi_0 |g - g_c|^{-\nu}$$

- The energy gap of $H(g)$ closes at the QPT

$$\Delta = \Delta_0 |g - g_c|^{z\nu}$$



→ Universality classes and critical exponents

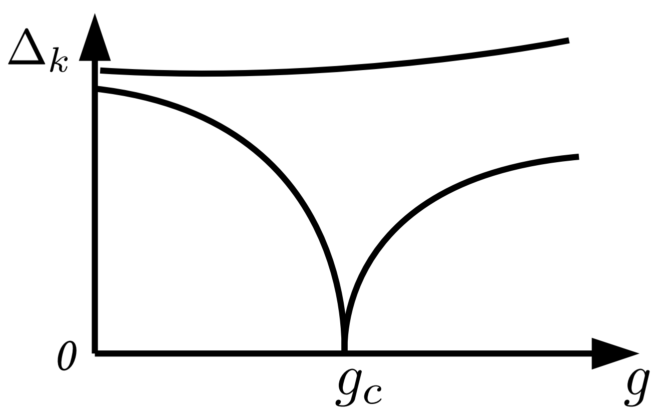
Our aim is to understand what happens when the system is driven to the QPT

Critical long-range interacting systems

- Short-range interacting system: One-dimensional transverse-field Ising model
  Many interesting critical phenomena: QPTs, dissipative & dynamical phase transitions, ...

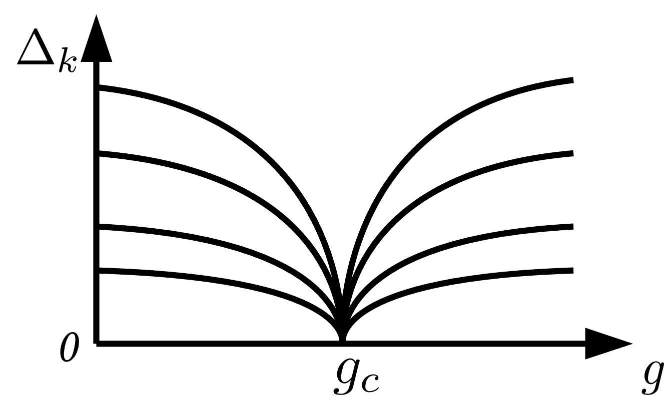
- Long-range (or fully-connected) interacting systems:

  - All-to-all interactions (isotropic and homogeneous couplings)
  - No notion of spatial dimension (d=0, zero-dimensional models)
  - Mean-field critical systems



short-range model

vs



mean-field system

A. Polkovnikov, K. Sengupta, A. Silva, M. Vengalattore, *Rev. Mod. Phys. 83, 863 (2011)*
J. Dziarmaga, *Adv. Physics, 59, 1063 (2010)*
J. Eisert, M. Friesdorf, C. Gogolin, *Nature Phys. 11, 124 (2015)*
M. Heyl, *Rep. Prog. Phys. 81, 054001 (2018)*

Critical long-range interacting systems

- **Lipkin-Meshkov-Glick model**:

$$H_{\mathrm{LMG}} = -\frac{1}{2N} \sum_{1\leq i<j\leq N} \sigma_i^x \sigma_j^x - \frac{g}{2} \sum_{i=1}^{N} \sigma_i^z$$

In the thermodynamic limit, a ferro-to-paramagnetic QPT takes place at *g=1*

- **Quantum Rabi model:**

$$H_{\mathrm{QRM}} = \omega a^\dagger a + \frac{\Omega}{2}\sigma_z + \lambda(a + a^\dagger)\sigma_x$$

In the limit $\Omega/\omega \to \infty$ there is a superradiant QPT at $\lambda_c = \sqrt{\omega\Omega}/2$

P. Ribeiro, J. Vidal, and R. Mosseri, *Phys. Rev. Lett.* ***99****, 050402 (2007)*
M.-J. Hwang, RP, and M. B. Plenio, *Phys. Rev. Lett.* ***115****, 180404 (2015)*

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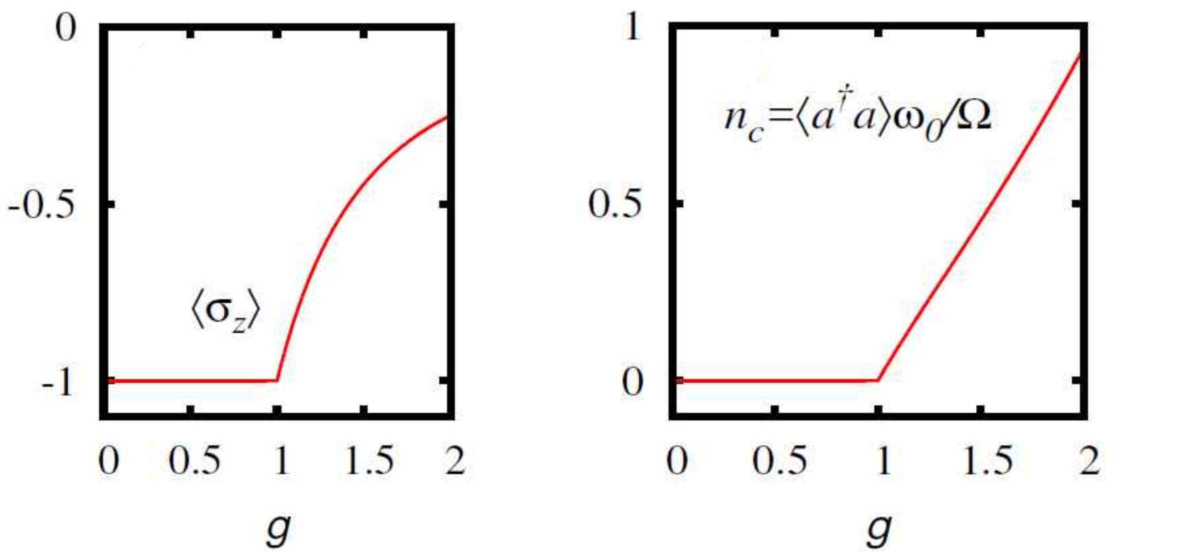

$g = \lambda/\lambda_c$

P. Ribeiro, J. Vidal, and R. Mosseri, *Phys. Rev. Lett.* ***99****, 050402 (2007)*
M.-J. Hwang, RP, and M. B. Plenio, *Phys. Rev. Lett.* ***115****, 180404 (2015)*

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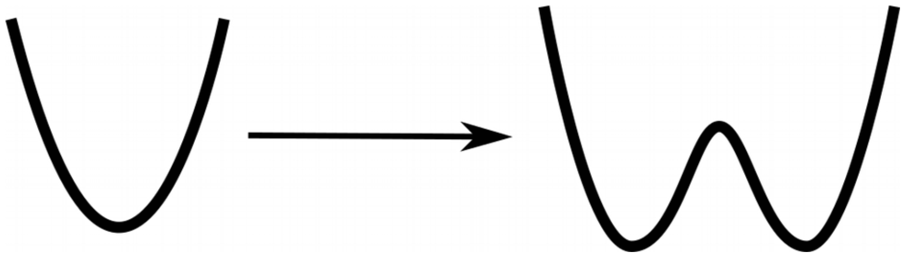
- Both models belong to the mean-field class:



P. Ribeiro, J. Vidal, and R. Mosseri, *Phys. Rev. Lett.* ***99***, *050402 (2007)*
M.-J. Hwang, RP, and M. B. Plenio, *Phys. Rev. Lett.* ***115***, *180404 (2015)*

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In the limit $\Omega/\omega \to \infty$ there is a superradiant QPT at $\lambda_c = \sqrt{\omega\Omega}/2$

- Both models belong to the mean-field class:

$$H_{\mathrm{eff}} = \omega a^\dagger a - \frac{g^2\omega}{4}(a+a^\dagger)^2 + \frac{f(g)}{\eta}(a+a^\dagger)^4$$

Rescaled coupling $g = \lambda/\lambda_c$

System size $\eta = \Omega/\omega$

$\eta = N$

P. Ribeiro, J. Vidal, and R. Mosseri, *Phys. Rev. Lett.* ***99****, 050402 (2007)*
M.-J. Hwang, RP, and M. B. Plenio, *Phys. Rev. Lett.* ***115****, 180404 (2015)*

Critical long-range interacting systems

- Critical features of these systems can be accessed/realized in experiments

Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator

J. Zhang[1], G. Pagano[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], H. Kaplan[1], A. V. Gorshkov[1], Z.-X. Gong[1]† & C. Monroe[1,2]

Selected for a Viewpoint in *Physics*

PRL **119,** 080501 (2017) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2017

Direct Observation of Dynamical Quantum Phase Transitions in an Interacting Many-Body System

P. Jurcevic,[1,2] H. Shen,[1] P. Hauke,[1,3] C. Maier,[1,2] T. Brydges,[1,2] C. Hempel,[1,*] B. P. Lanyon,[1,2] M. Heyl,[4,5] R. Blatt,[1,2] and C. F. Roos[1,2]

[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften,*

nature COMMUNICATIONS

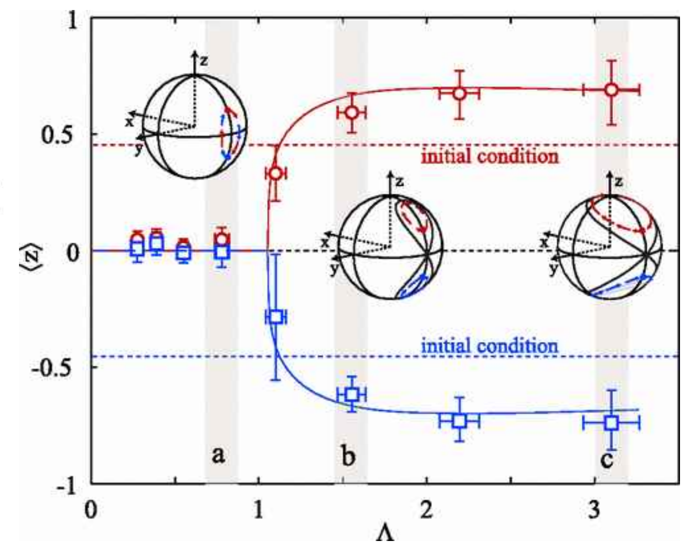
ARTICLE

https://doi.org/10.1038/s41467-021-21425-8 OPEN

Check for updates

Observation of a quantum phase transition in the quantum Rabi model with a single trapped ion

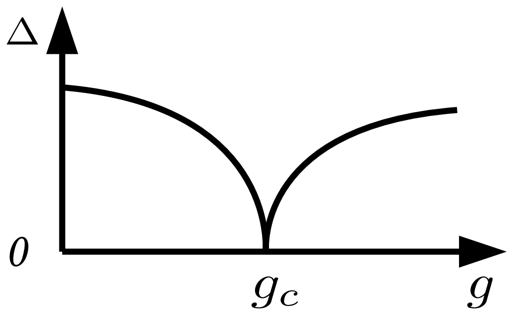
M.-L. Cai[1,4], Z.-D. Liu[1,4], W.-D. Zhao[1,4], Y.-K. Wu[1], Q.-X. Mei[1], Y. Jiang[1], L. He[1], X. Zhang[1,2], Z.-C. Zhou[1,3] & L.-M. Duan[1]✉

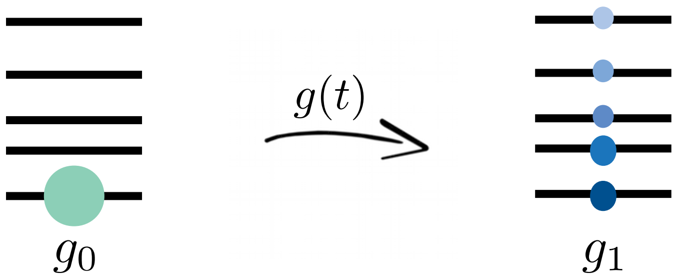


T. Zibold, E. Nicklas, C. Gross, M. K. Oberthaler
Phys. Rev. Lett. 105, 204101 (2010)

Nonequilibrium critical dynamics

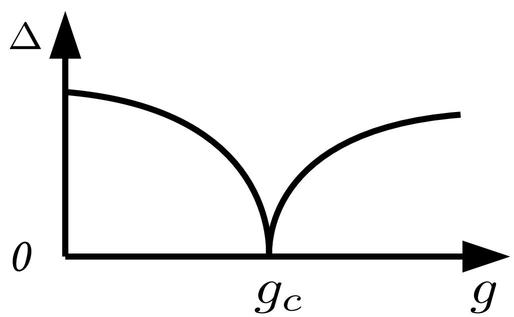
- Nonequilibrium dynamics triggered by a quantum critical point: *g*(t)

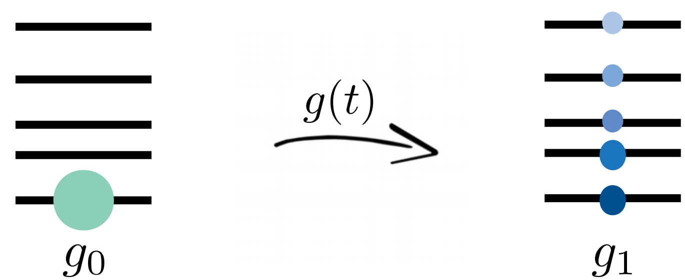




Nonequilibrium critical dynamics

- Nonequilibrium dynamics triggered by a quantum critical point: $g(t)$





- **Kibble-Zurek mechanism:** Adiabatic-Impulse

Relaxation time:

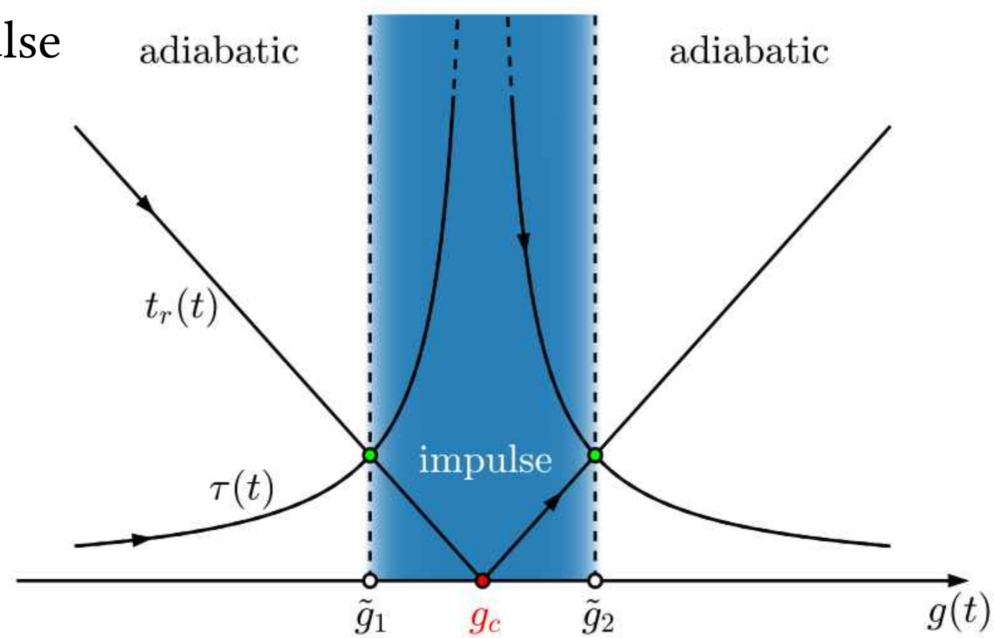
$$\tau = \tau_0 |g - g_c|^{-z\nu}$$

Timescale of changes in the system:

$$t_r(g) = |\Delta / \dot{\Delta}| \approx \tau_q |g - g_c|$$

Transition between adiabatic and impulsive:

$$\tau(\tilde{g}) = t_r(\tilde{g}) \Rightarrow |\tilde{g} - g_c| \sim \tau_q^{-1/(z\nu+1)}$$

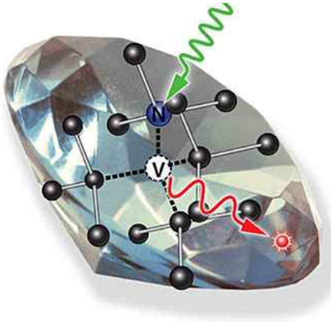


$$g(t) = g_0 + (g_1 - g_0)\frac{t}{\tau_q}$$

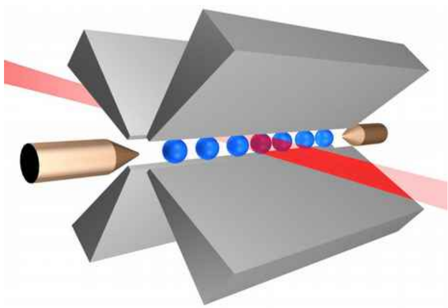
A. del Campo, W. H. Zurek, *Int. J. Mod. Phys. A 29, 1430018 (2014)*

Quantum metrology

- Quantum metrology: Potential enhancement in parameter estimation when using quantum resources (quantum correlations, entangled states, quantum light, etc.)



NV center



Trapped ions

- Quantum Fisher Information (QFI): $$I_\omega = 4\left[\langle \partial_\omega \psi | \partial_\omega \psi \rangle + \left(\langle \partial_\omega \psi | \psi \rangle\right)^2\right]$$

- Cramer-Rao bound: $$\delta\omega \geq \frac{1}{\sqrt{I_\omega}}$$

M. G. A. Paris, *Int. J. Quant. Inf.* ***7***, *125 (2009)*

Quantum metrology

- The QFI sets the ultimate precision limit in parameter estimation:

  Standard Quantum limit (**classical**): $I_\omega \sim NT$ $\delta\omega \sim 1/\sqrt{NT}$

  Heisenberg scaling or limit (**q-resources**): $I_\omega \sim N^2T^2$ $\delta\omega \sim 1/(NT)$

- Critical systems are very sensitive to variations close to the phase transition
  QPTs appear as a valuable resource for quantum metrology!

- The QFI diverges at the QPT $I_\omega \propto |g - g_c|^{-2}$

- No free lunch theorem: *Infinite precision requires infinite time*

What if we do not prepare the ground state but simply approach the QPT?
What is the scaling of the QFI in this case?

M. G. A. Paris, *Int. J. Quant. Inf.* ***7***, *125 (2009)*
V. Giovannetti, S. Lloyd, and L. Maccone, *Phys. Rev. Lett.* ***96***, *010401 (2006)*
J. J . Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heizen, *Phys. Rev. A* ***54***, *R4649(R) (1996)*

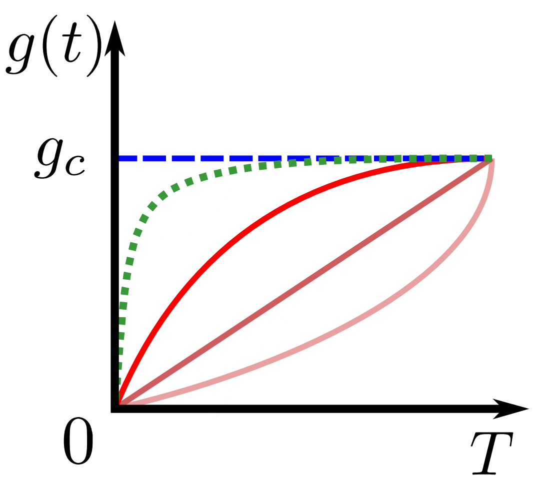
Dynamical metrological protocol

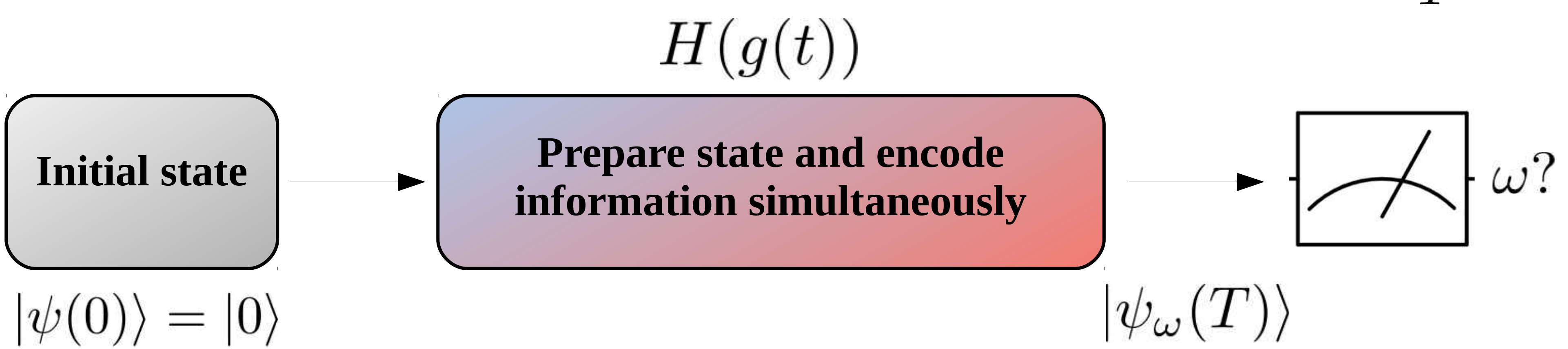
- Effective model in the thermodynamic limit:

$$H_{\text{eff}} = \omega a^\dagger a - \frac{g^2\omega}{4}(a + a^\dagger)^2$$

- Finite-time (nonlinear) ramp towards the QPT:

$$g(t) = 1 - \left(\frac{T - t}{T}\right)^r$$

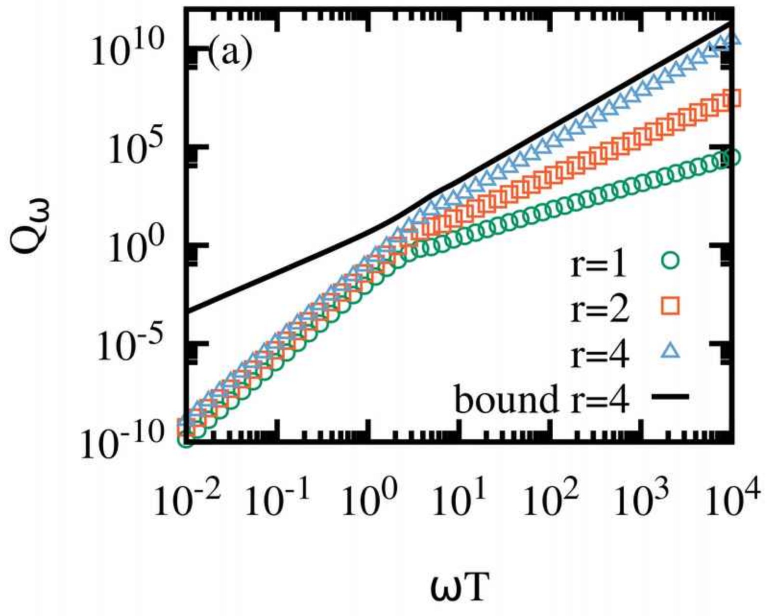




L. Garbe, O. Abah, S. Felicetti, RP, *Quantum Sci. Technol. 7, 035010 (2022)*

Finite-time ramp

$$Q_\omega = \omega^2 I_\omega$$



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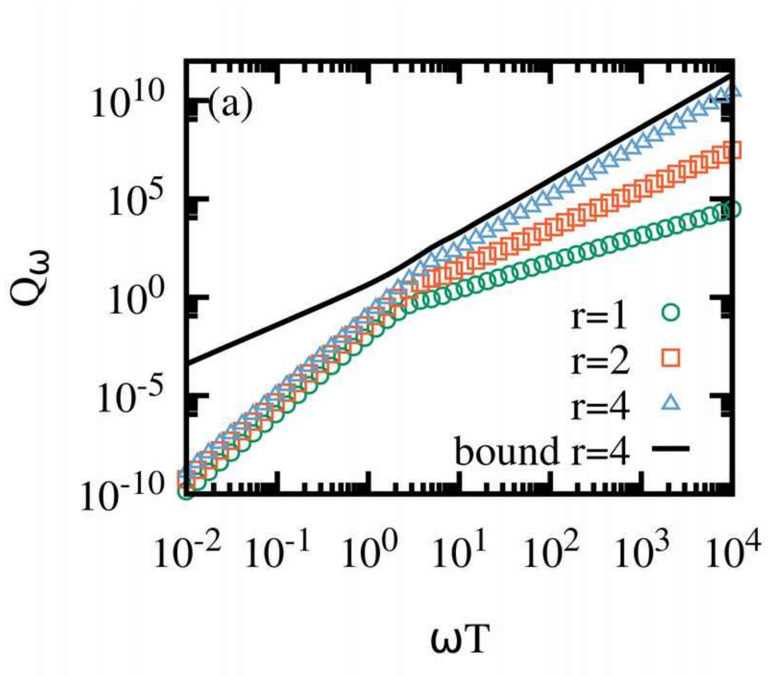
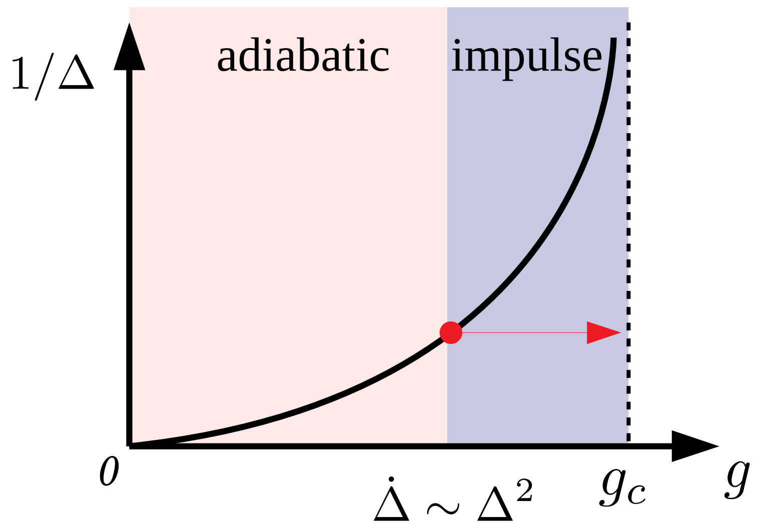




$$g_f \approx 1 - (T\omega/r)^{-2r/(2+r)}$$

$$I_\omega(g_c) \approx I_\omega(g_f) \Rightarrow I_\omega \propto (\omega T/r)^{4r/(2+r)}$$

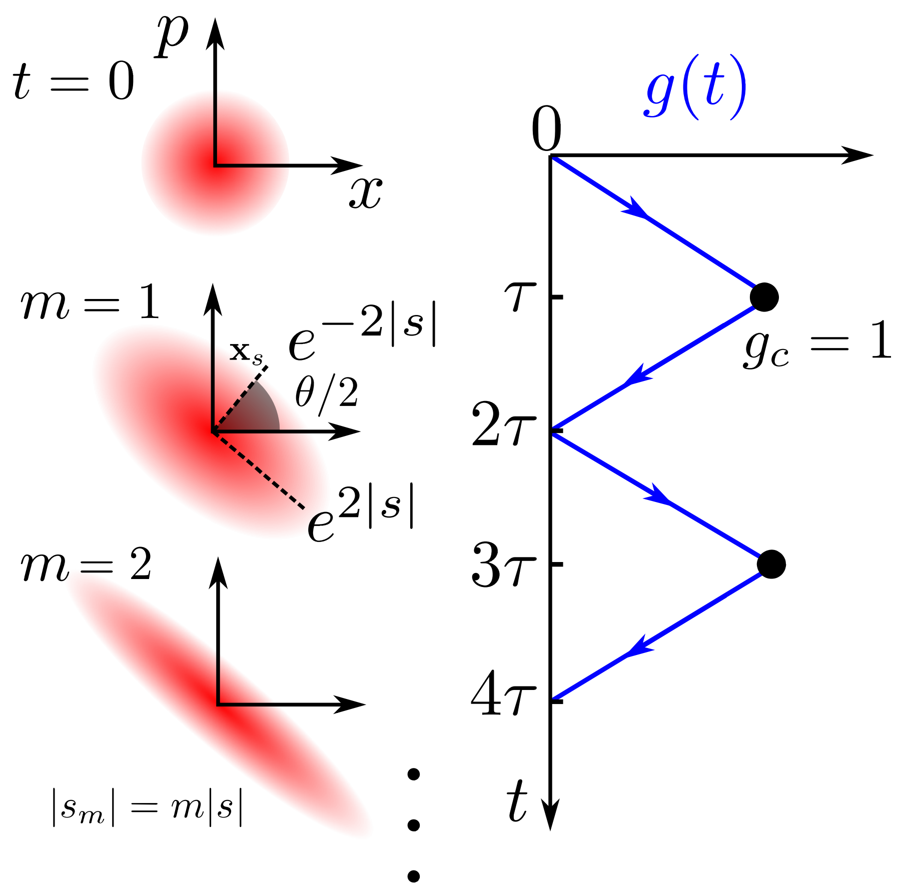
The QFI scales as dictated by the Kibble-Zurek mechanism...but what about Heisenberg?

This coincides with the Heisenberg limit $I_\omega \sim N^2 T^2$ since the *N* (number of bosonic excitations) also scales with time

Kibble-Zurek scaling = Heisenberg scaling

L. Garbe, O. Abah, S. Felicetti, RP, *Quantum Sci. Technol. 7, 035010 (2022)*

Can we do better?



$$H_{\text{eff}} = \omega a^\dagger a - \frac{g^2(t)\omega}{4}(a + a^\dagger)^2$$

$$|\psi(2\tau)\rangle = S(s)|0\rangle$$

$$|s| = \log(3)/2 \qquad \omega\tau \gtrsim 1$$

Repeating the cycle *m* times

$$\rightarrow |s_m| = m|s| = m\log(3)/2$$

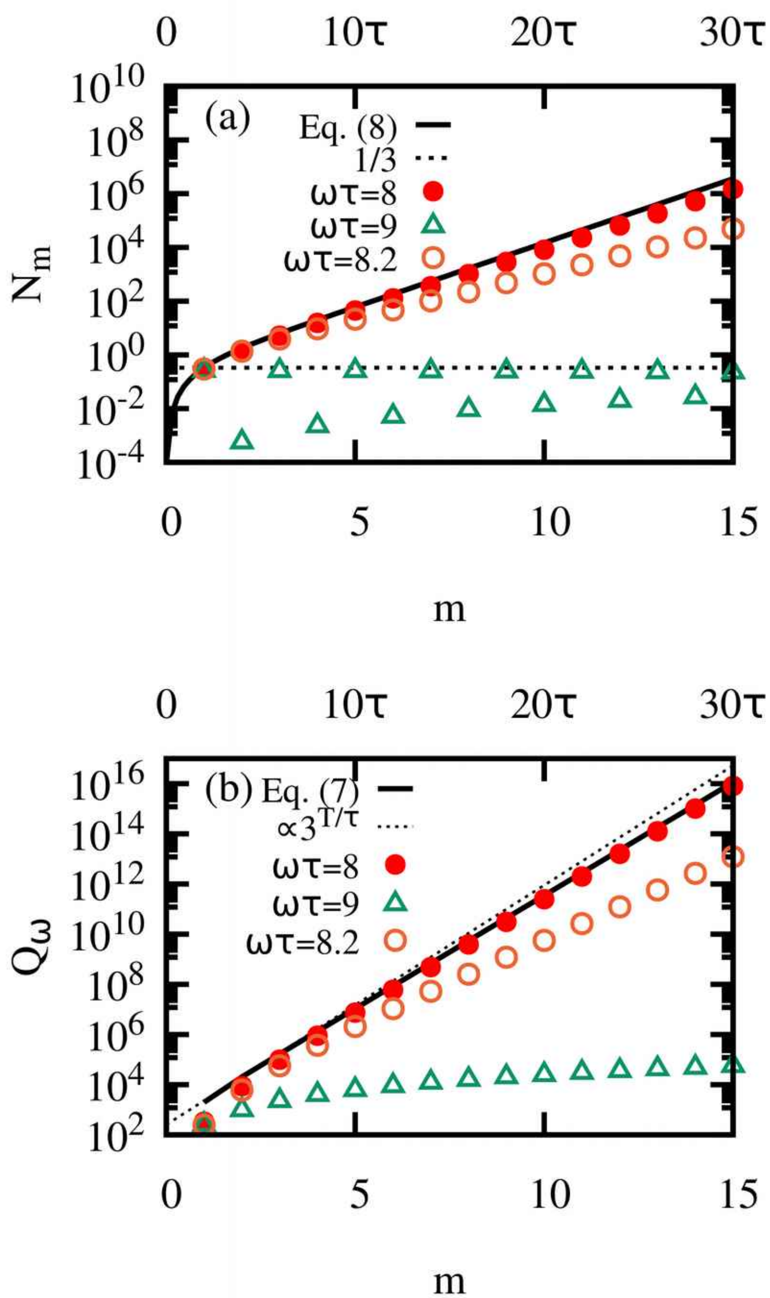
which requires a phase-matching condition

$$\theta_{m+1} = \theta_m \qquad \omega\tau = 2n, \quad n = 1, 2, \ldots$$

$$N(2\tau m) = \sinh^2(m\log(3)/2) \sim 3^m$$

L. Garbe, O. Abah, S. Felicetti, RP, *Phys. Rev. Research 4, 043061 (2022)*

Exponential-time precision



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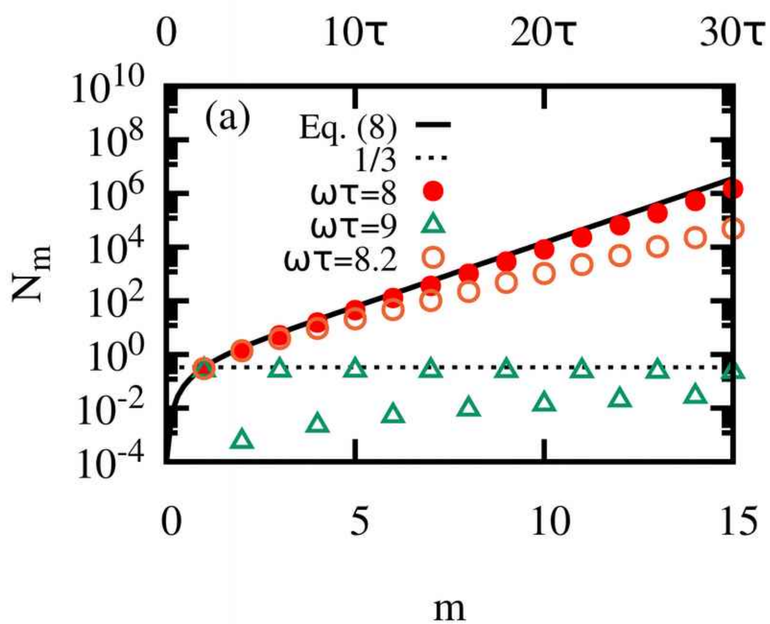
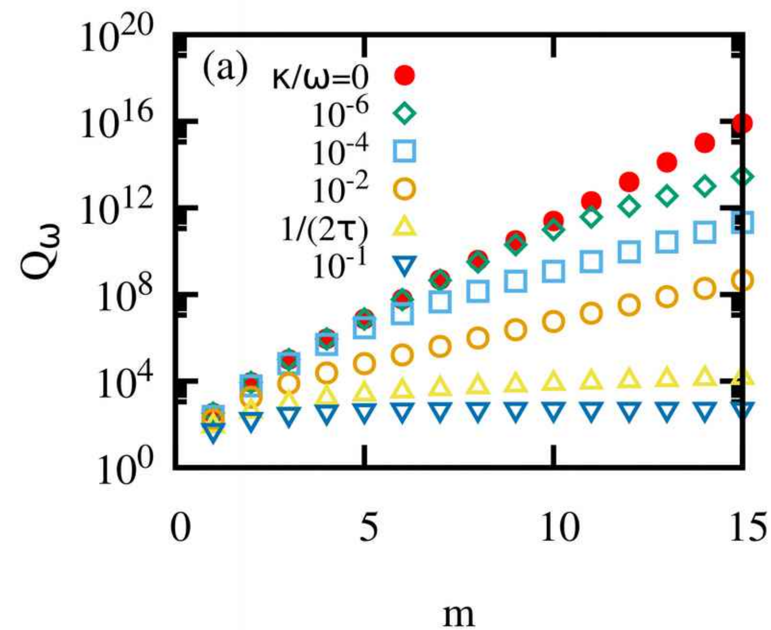
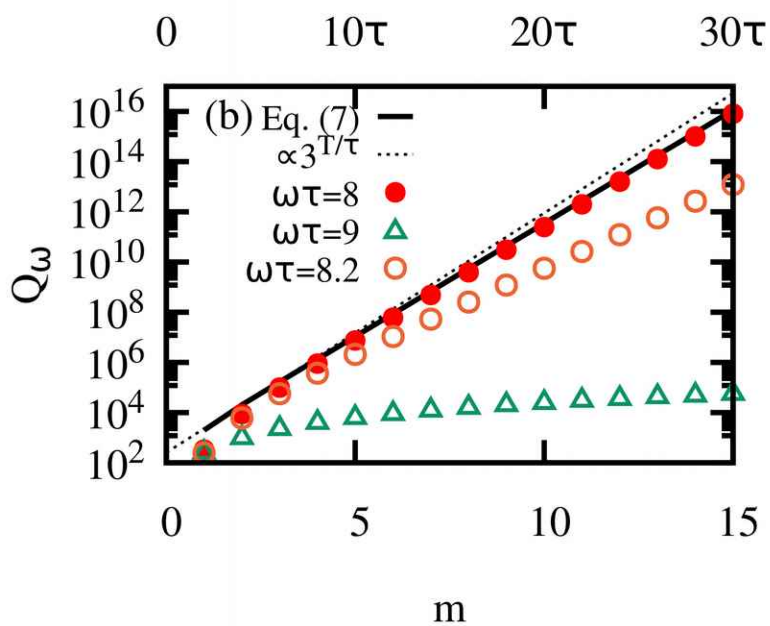
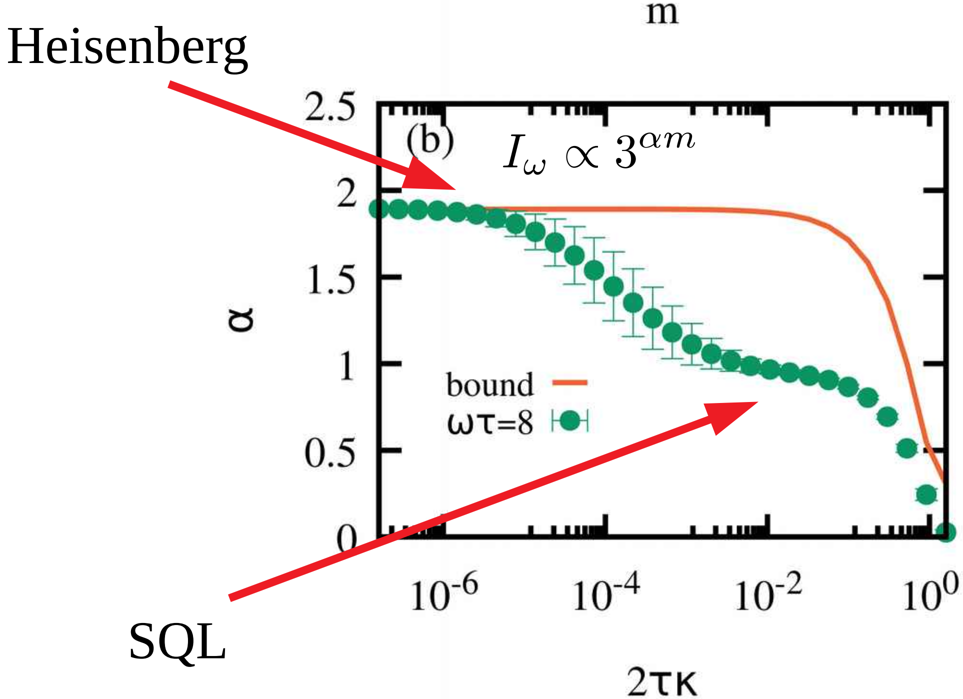








Quantum thermodynamics

- Understand nonequilibrium dynamics from a thermodynamic perspective (e.g. Work statistics)

- Critical phenomena have been linked to the emergence of irreversible dynamics at the quantum level:
  How is irreversible work and irreversible entropy in these systems?

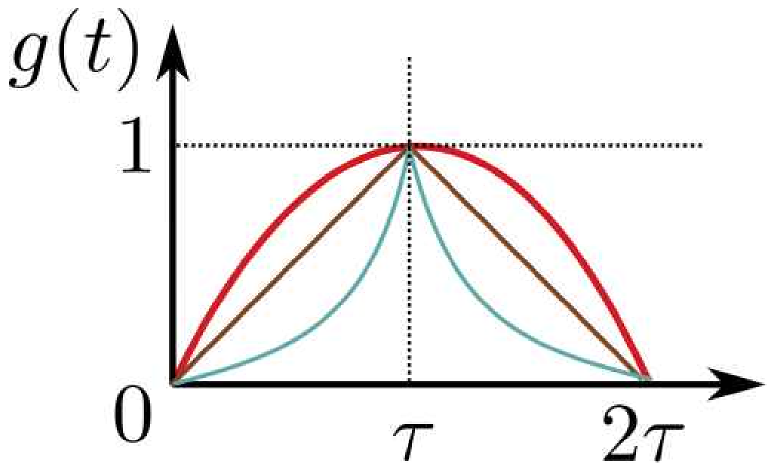
- Understand the role of quantum coherence in the irreversible entropy

- Finite-time cycles involving the QPT:

$$H_{\text{eff}} = \omega a^\dagger a - \frac{g^2\omega}{4}(a + a^\dagger)^2$$

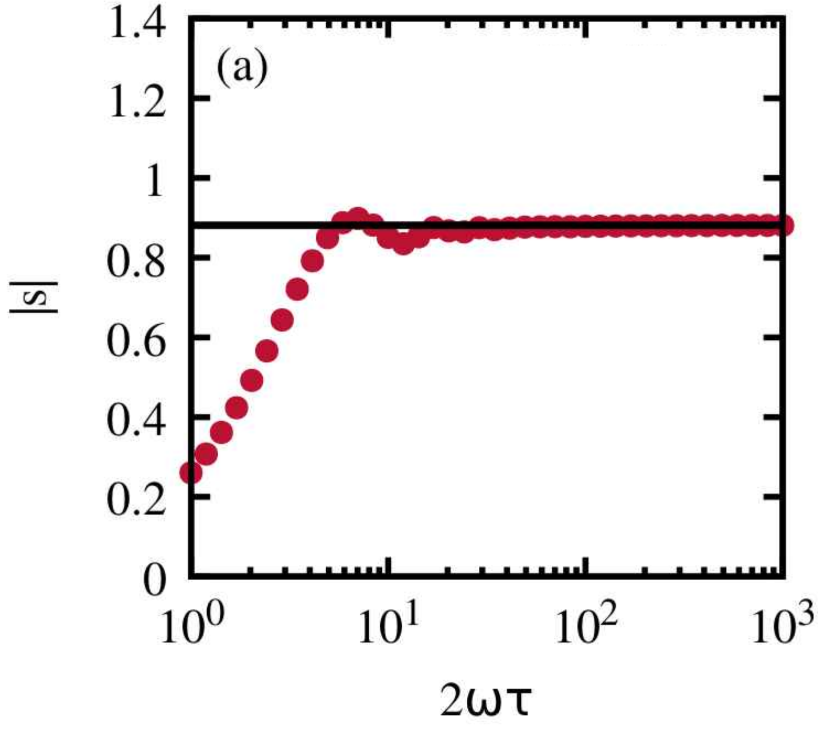
$$\langle W_{\text{irr}} \rangle = \langle W \rangle - \Delta F$$

$$\Delta F = 0$$



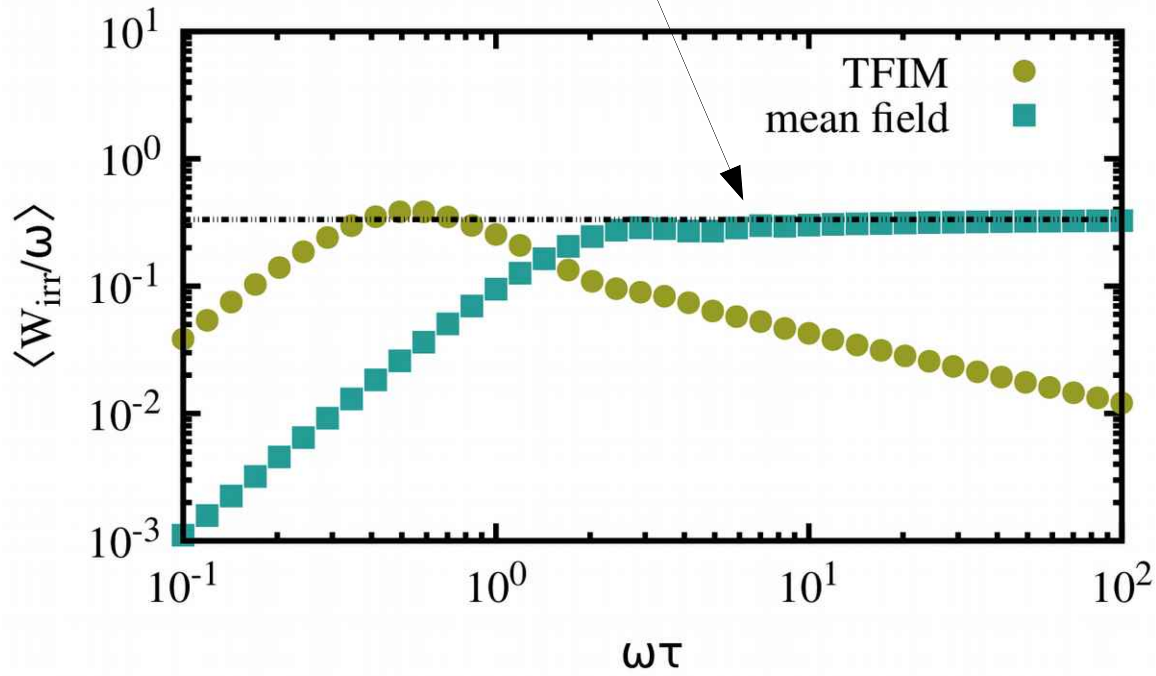
R. Dorner, J. Goold, C. Cormick, M. Paternostro, V. Vedral, *Phys. Rev. Lett. 109, 160601 (2012)*
A. Varizi, A. P. Viera, C. Cormick, R. C. Drumond, G. T. Landi, *Phys. Rev. Research 2, 033279 (2020)*

Irreversible work



$$|s| = \mathrm{acosh}\left[\csc\left(\frac{\pi}{2+2z\nu r}\right)\right]$$

Irreversible work saturates even for infinitely slow ramps



Breakdown of the adiabatic condition leading to Kibble-Zurek scaling:

$$\langle W_{\mathrm{irr}} \rangle \sim \tau^{-d\nu r/z\nu r+1}$$

F. J. Gomez-Ruiz, S. Gherardini, RP *(in preparation)*

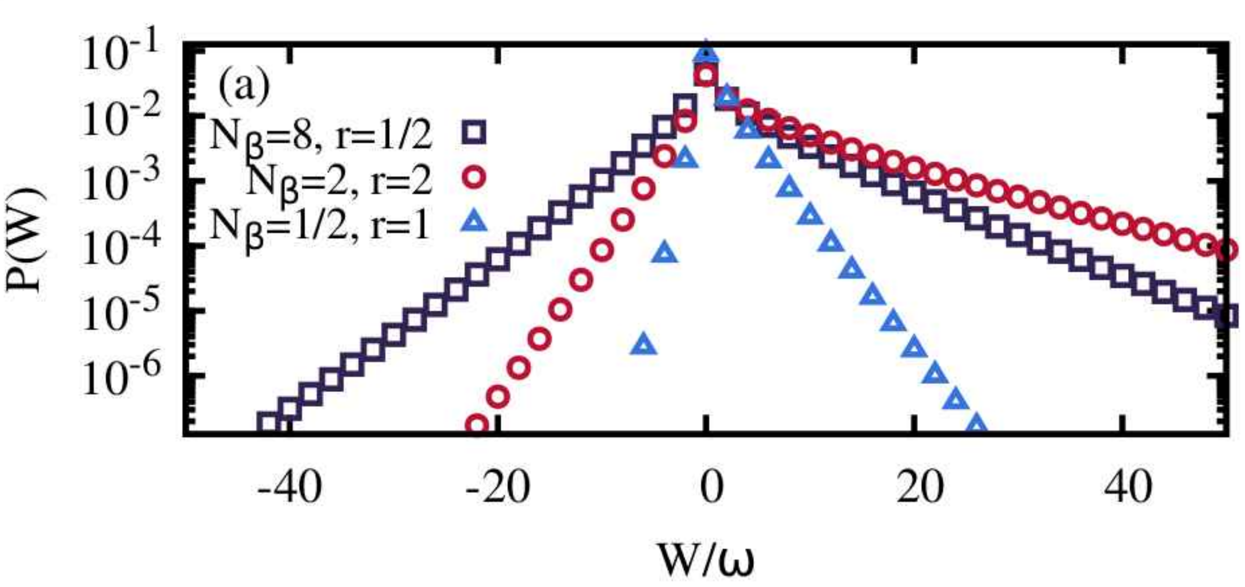
Work statistics

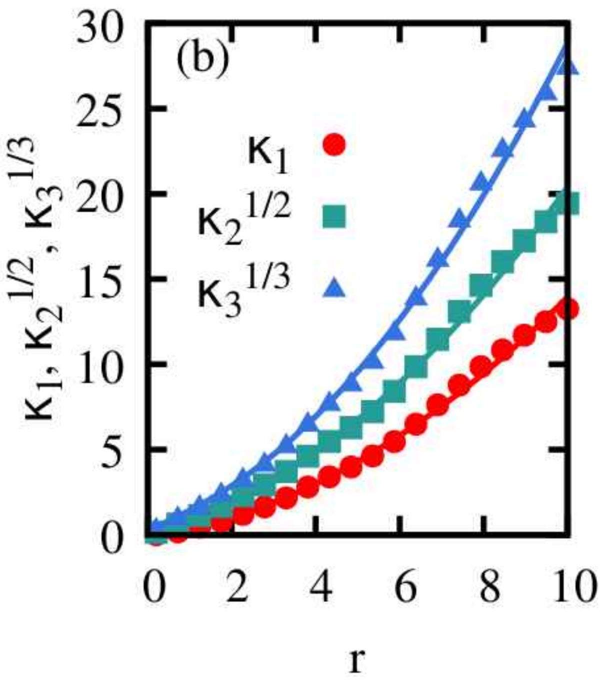
- Initial thermal state ⟶ squeezed thermal state

$$\rho_\beta \to \mathcal{S}(s)\rho_\beta\mathcal{S}^\dagger(s)$$

- Work statistics (two-point measurement scheme) can be completely characterized:

$$P(W) = \sum_{n,m} p_n^0 p_{m|n}^{2\tau} \delta[W - (E'_m - E_n)]$$





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Irreversible entropy

- Irreversible entropy can be completely worked out

$$\langle S_{\text{irr}} \rangle = \beta \langle W_{\text{irr}} \rangle = \beta\omega \coth\left[\frac{\beta\omega}{2}\right] \cot^2\left[\frac{\pi}{2(1+z\nu r)}\right]$$

- Irreversible entropy can be divided in two contributions:

$$\langle S_{\text{irr}} \rangle = D + C$$

D: changes in populations

C: changes in coherence

$$C = S_v(\Gamma[\hat{\rho}(2\tau)]) - S_v(\hat{\rho}(2\tau))$$ Relative entropy of coherence

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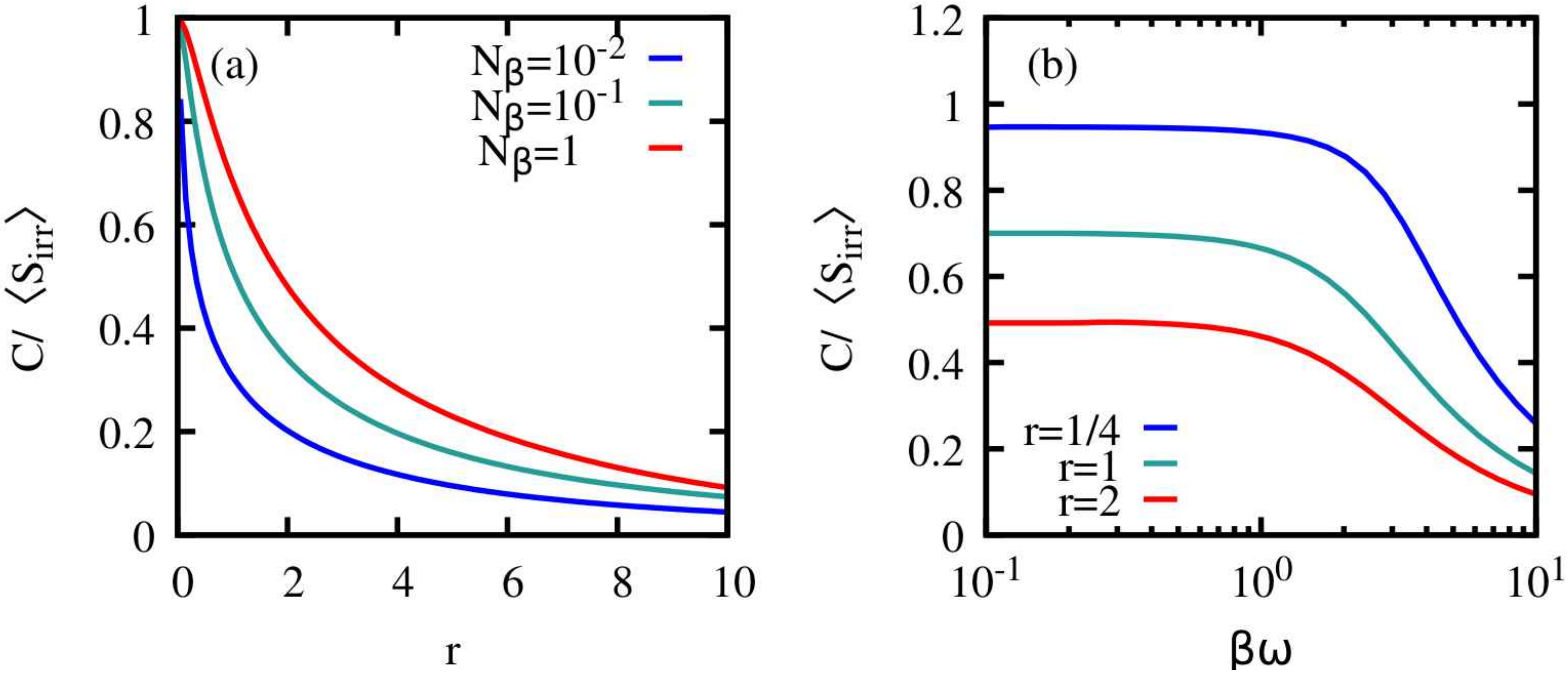

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Conclusions

- Systems undergoing a quantum phase transition are a valuable resource for quantum metrology and to explore novel phenomena in quantum thermodynamics

- **Universal scaling regimes** for the precision in long-range systems (Kibble-Zurek = Heisenberg) which can lead to an **exponential scaling** under periodic modulation

- Finite-time cycles lead to a **saturation of irreversible work, universal work statistics**, where quantum coherence may be responsible for most of the irreversible entropy

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Obrigado!

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