Superconducting circuits: from photon generation to universal quantum gates

Fernando C. Lombardo

Departamento de Física
.UBAexactas

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Collaboration

Reference: arXiv:2408.08404v1

A controlled-squeeze gate in superconducting quantum circuits

Nicolás F. Del Grosso,[1] Rodrigo G. Cortiñas,[2] Paula I. Villar,[1] Fernando C. Lombardo,[1] and Juan Pablo Paz[1]

[1] *Departamento de Física Juan José Giambiagi, FCEyN UBA and IFIBA UBA-CONICET, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

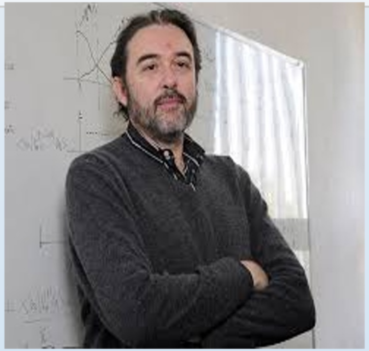
[2] *Department of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA*

We present a method to prepare non-classical states of the electromagnetic field in a microwave resonator. It is based on a controlled gate that applies a squeezing operation on a SQUID-terminated resonator conditioned on the state of a dispersively coupled qubit. This controlled-squeeze gate, when combined with Gaussian operations on the resonator, is universal. We explore the use of this tool to map an arbitrary qubit state into a supersposition of squeezed states. In particular, we target a bosonic code with well-defined superparity and photon loss is thus error detectable by nondemolition parity measurements. We analyze the possibility of implementing this using state-of-the-art circuit QED tools and conclude that it is within reach of current technologies.

Nicolás del Grosso

Paula Villar

Fernando Lombardo



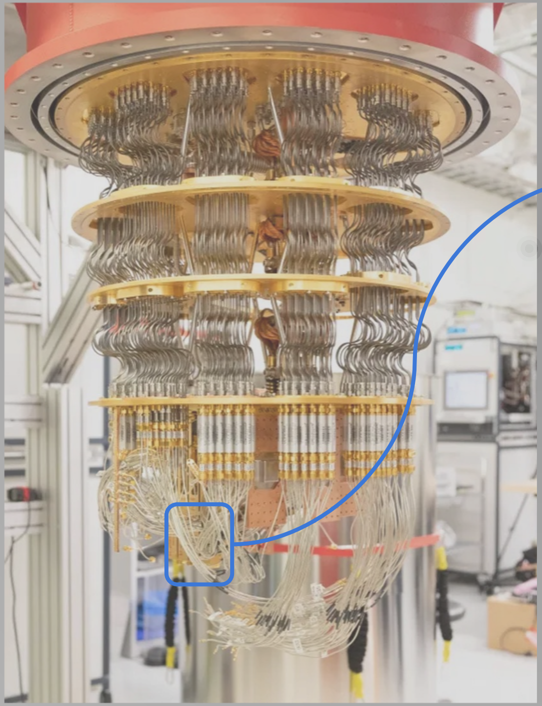
Juan Pablo Paz

Rodrigo Cortiñas

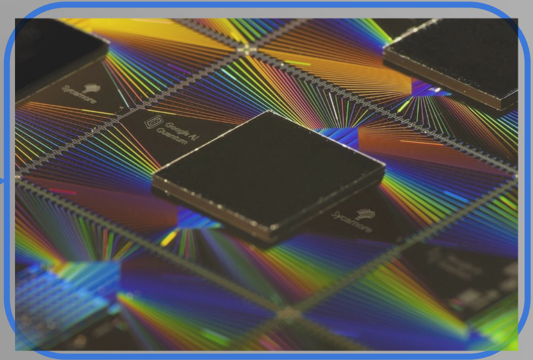
Why Circuit Quantum Electrodynamics?

Circuit Quantum Electrodynamics

Promising Platform for successful Quantum Computation



Sycamore Google Quantum Computer



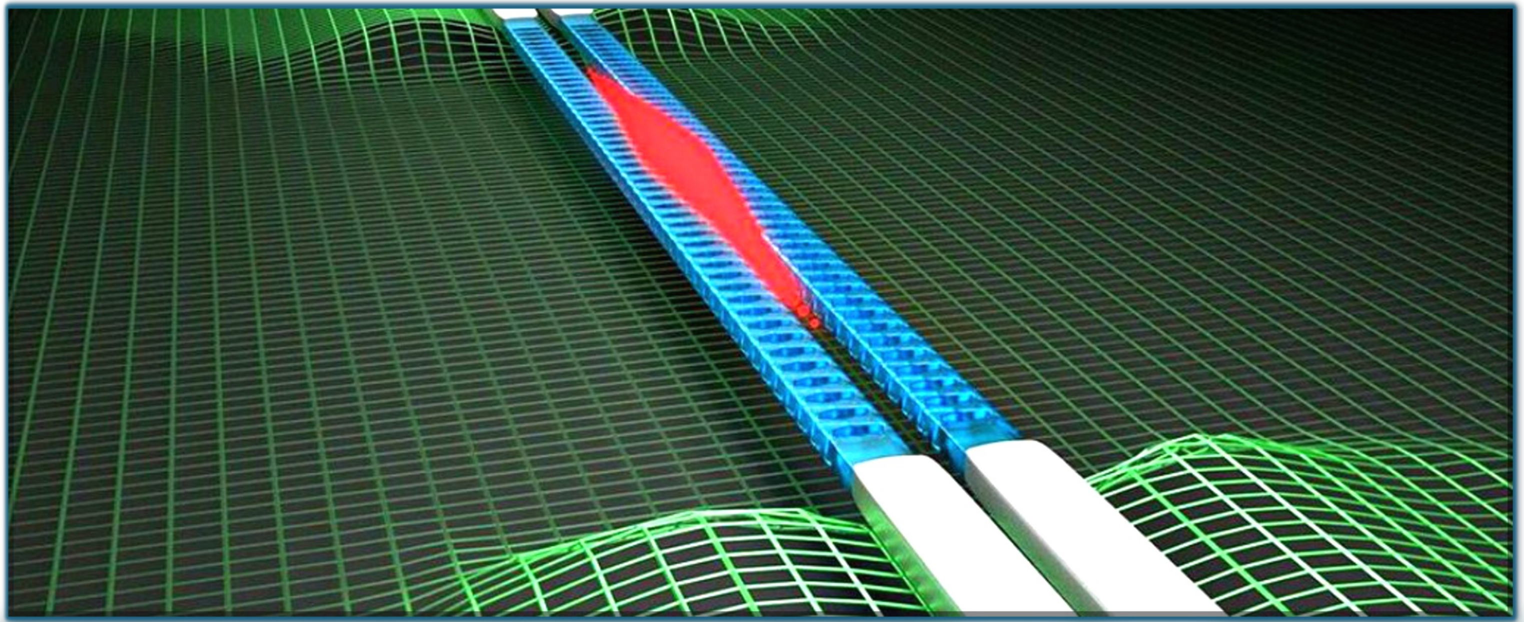
Photograph of the Sycamore chip.

~~QUANTUM SUPREMACY~~

2019: 53 qubits
2024: 80 qubits
2025: 1000 qubit

QUANTUM UTILITY

Circuit Quantum Electrodynamics has also provided the Simulation of the Dynamical Casimir Effect



Very difficult to observe…

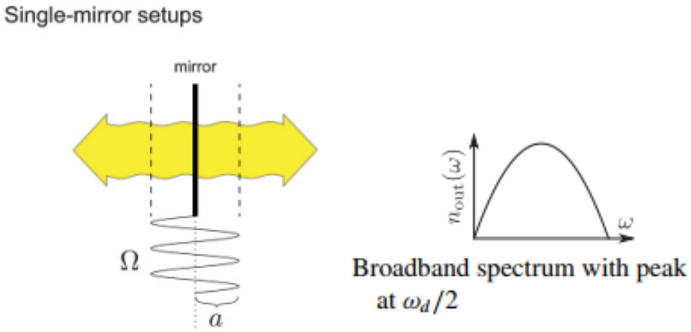
Rate of photon production by a single oscillating mirror *in vacuum*

$$\frac{N}{T} = \frac{A}{60\pi^2}\frac{\Omega^3}{c^2}\left(\frac{v_{max}}{c}\right)^2$$

$V = \Omega \, . \, a$

$$\frac{v_{max}}{c} = 10^{-7} \quad \Omega = 10GHz \quad A = 10cm^2$$

1 photon/day!!



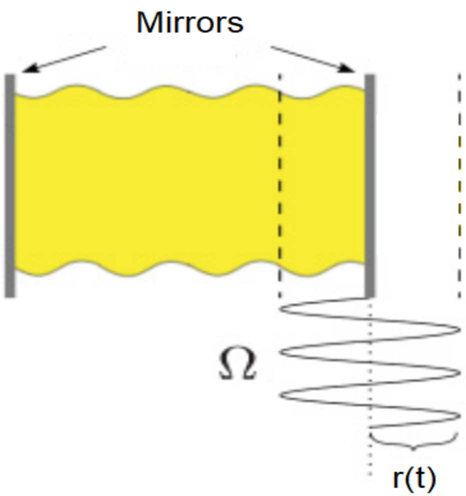
[1]G.T. Moore, J. Math. Phys. 11, 2679 (1970)
[2]A. Lambrecht, M.T. Jaekel and S. Reynaud, Phys. Rev. Lett.77, 615 (1996)

For *cavities* the situation is better due to parametric resonance… but still difficult

- In order to produce 5 GHz photons, we need mechanical oscillations with 10GHz
- Actual limit: 6GHz

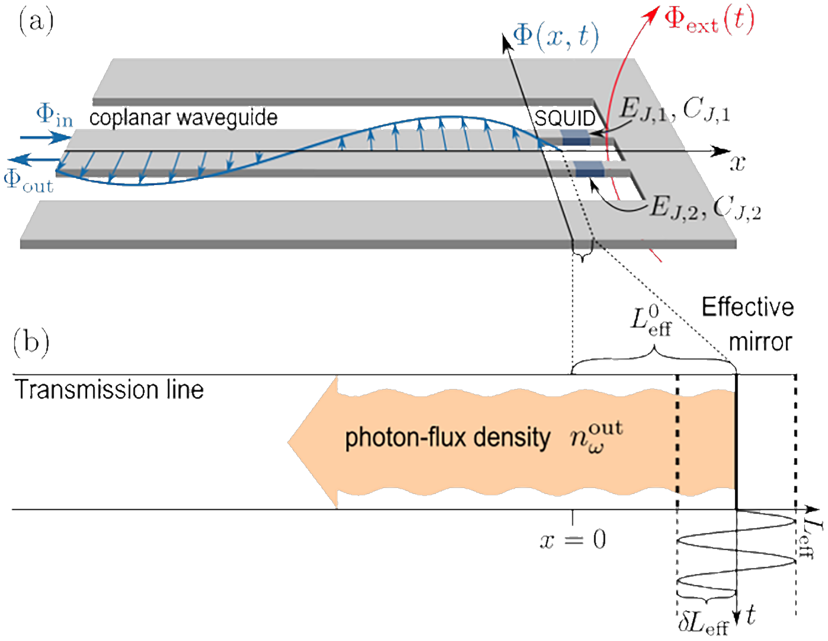
$$N = e^{\eta\epsilon\Omega t}, \eta = O(1)$$

$$N_{max} \simeq e^{\epsilon Q} \leq e^{10^{-8}Q}$$



EXPERIMENTAL VERIFICATION OF DCE (2011)

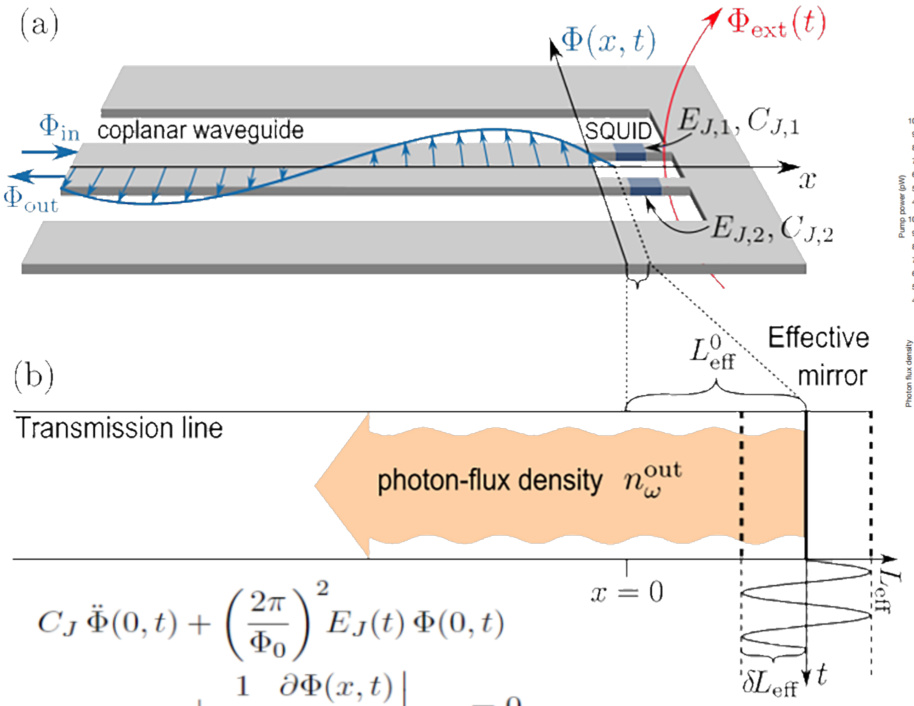
By applying a time-dependent magnetic flux through the SQUID we get a time-dependent inductance, which in turn produces a time-dependent boundary condition for the field in the waveguide



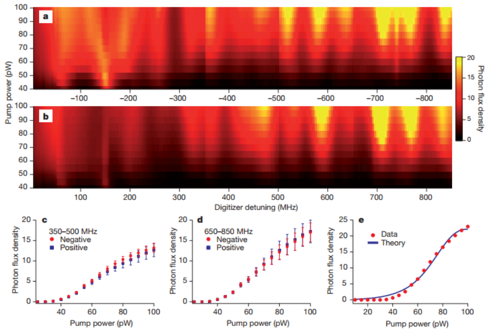
Modulated inductance of SQUID at high frequencies (> 10 GHz)

[4] J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, Phys. Rev. Lett. 103, 147003 (2009)

[5] G. Johansson, A. Pourkabirian, M. Simoen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Nature 479, 376 (2011)

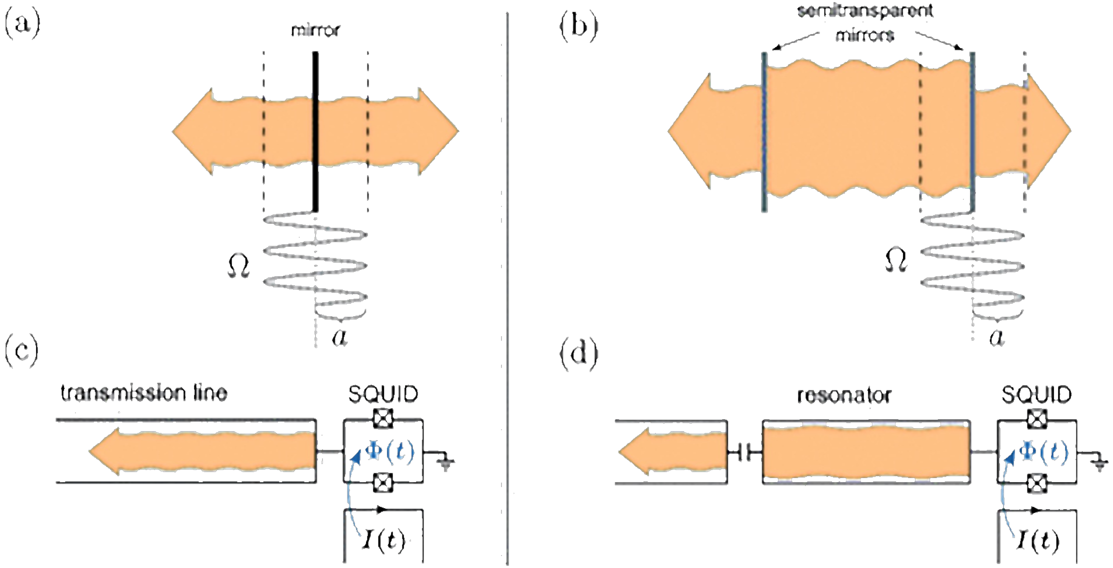


$$C_J\,\ddot{\Phi}(0,t) + \left(\frac{2\pi}{\Phi_0}\right)^2 E_J(t)\,\Phi(0,t) + \frac{1}{L_0}\,\frac{\partial\Phi(x,t)}{\partial x}\bigg|_{x=0} = 0$$



Time dependent boundary condition

DIFFERENT EXPERIMENTAL REALIZATIONS OF DCE



J. R. Johansson, G. Johansson, C. M. Wilson, and F. Nori, Phys. Rev. A 82, 052509 (2010).

Outline

1. Introduction to Circuit Quantum Electrodynamics

2. New Universal Gate : Controlled-squeeze gate

3. What can we do with it? Encoding quantum states in the resonator in an error-detectable way

4. Summary

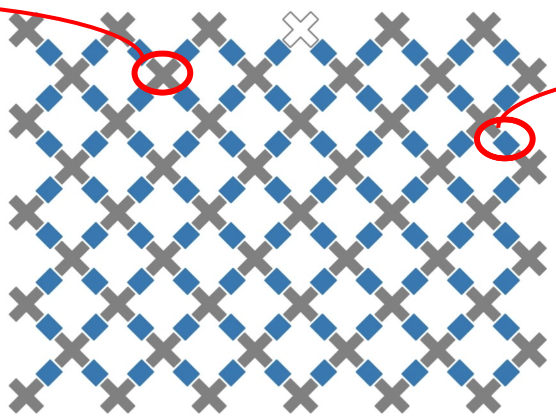
1. Introduction to Circuit Quantum Electrodynamics

1. Introduction to Circuit Quantum Electrodynamics

Circuit QED: quantum circuits with quantum atoms and resonators interconnecting them

Artificial atoms

Two level atoms (Qubits) made with superconducting elements

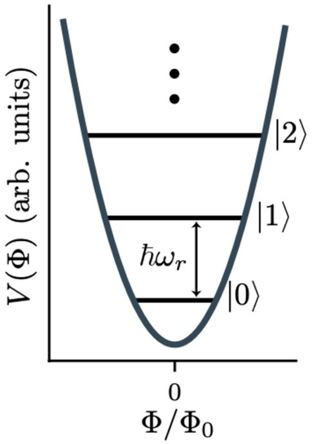


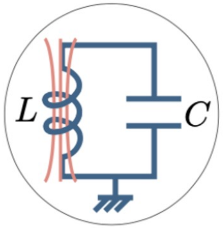
(rf) Resonators

Waveguide storing single mode of the EM field: Quantum oscillator

Scheme of a typical circuit QED setup

LC circuit: the simplest electronic resonator





$$\omega_r = \frac{1}{\sqrt{LC}}$$

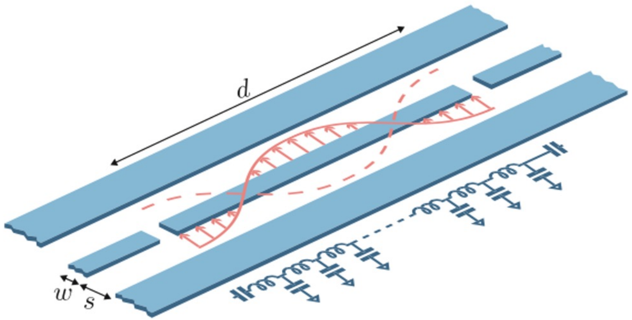
$$\hat{H}_{\mathrm{LC}} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} \qquad [\hat{\Phi}, \hat{Q}] = i\hbar.$$

$$\hat{H}_{\mathrm{LC}} = \hbar\omega_r \hat{a}^\dagger \hat{a}$$

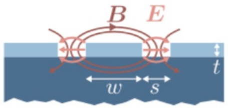
Quantum harmonic oscillators come in many shapes and sizes and constitute different elements of a circuit

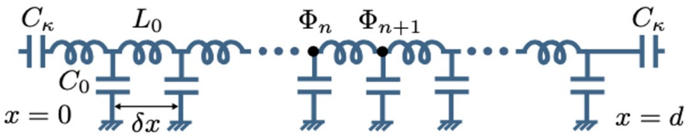
Resonator or Transmission line

Storages excitations of the electromagnetic field



Series of LC-Resonators





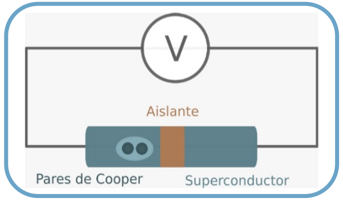
$$H = \sum_{n=0}^{N-1} \left[\frac{1}{2C_0} Q_n^2 + \frac{1}{2L_0} (\Phi_{n+1} - \Phi_n)^2 \right].$$

<u>One mode approximation:</u> $\hat{H}_r = \hbar \omega_r \hat{a}^\dagger \hat{a}$

1. Introduction to Circuit Quantum Electrodynamics

Artificial atom

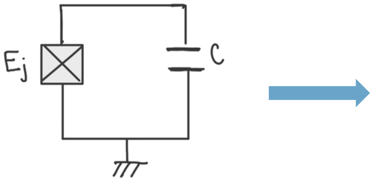
Based on a Josephson Junction

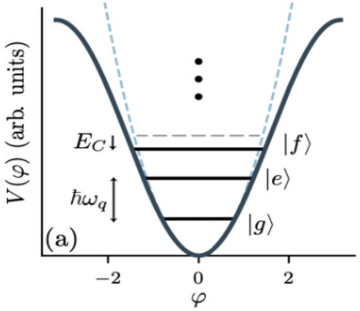


$$I = I_C \sin\left(\frac{2\pi\Phi}{\Phi_0}\right)$$

We shall use a transmon (most stable artificial atom)

Non linear Inductance





Non linear potential well of the transmon qubit (full line) compared to the quadratic potential of the LC oscillator (dashed lines).

$$\hbar\omega_q = \hbar\omega_q(E_C, E_J)$$

$$\hat{H}_q = \frac{\hat{Q}^2}{2C_\Sigma} - \frac{\Phi_0}{2\pi} I_c \cos\left(\frac{2\pi}{\Phi_0}\hat{\Phi}\right)$$

$$\hat{H}_q = 4E_C\,\hat{n}^2 - E_J \cos(\hat{\varphi})$$

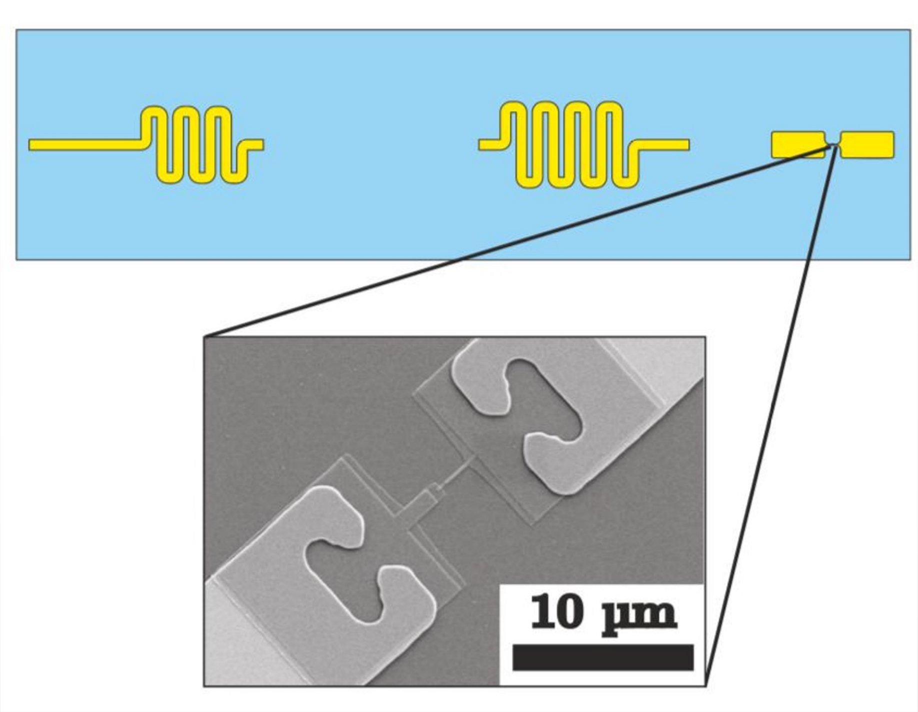
⋮

$$E_J/E_C >> 1$$

$$\hat{H}_q = \frac{\hbar\omega_q}{2}\hat{\sigma}_z = \frac{\hbar\omega_q}{2}\Big(\,|0\rangle\langle 0| - |1\rangle\langle 1|\,\Big)$$

In reality, things do not look like in paper

This is a transmon



Interaction qubit-resonator

$$H_{\text{int}} = \frac{C_\chi}{2}(\Delta V)^2$$

$$\hat{H}_{\text{int}} = \frac{C_\chi}{2}\left(\frac{\hat{Q}_{\text{trans}}}{C_J} - \frac{\hat{Q}_{\text{LC}}}{C}\right)^2$$

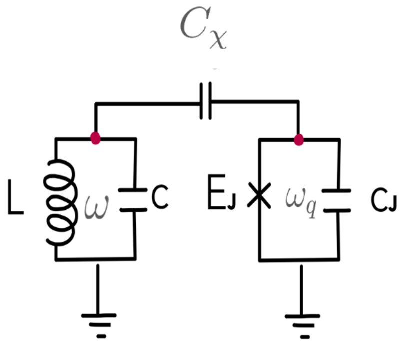
$$\hat{H}_{\text{int}} = \frac{C_\chi}{2}\left(\frac{\hat{Q}_{\text{trans}}\hat{Q}_{\text{LC}}}{C\ C_J}\right) + \dots$$

$$\vdots$$

In the one-mode approximation for the resonator and the 2-Level system for the transmon, using RWA

$$\hat{H}_{\text{int}} = \chi(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+)$$

$$\hat{H}_{\text{int}} = \chi a^\dagger a \sigma_z$$



$$\begin{cases}\hat{Q}_{LC} \rightarrow (\hat{a}^\dagger - \hat{a}) \\ \hat{Q}_{\text{trans}} \rightarrow (\hat{b}^\dagger - \hat{b})\end{cases}$$

$$\hat{b}^\dagger \rightarrow \hat{\sigma}_+$$

$$\hat{b} \rightarrow \hat{\sigma}_-$$

This is a Jaynes Cummings interaction Hamiltonian

$$\hat{H}_T = \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \chi\hat{a}^\dagger\hat{a}\hat{\sigma}_z$$

Resonant Case $\omega_r = \omega_q$

We shall work in the non resonant case $\omega_r \neq \omega_q$

Summary cQED Toolbox

- ➢ Qubits can be tunable (Google) and non tunable (IBM)
- ➢ Single qubit operation: rf pulses
- ➢ Gaussian operations on the resonator. Qubit-qubit interaction mediated by resonator. JC resonant, non resonant, strong coupling regime
- ➢ Universal set of gates: Controlled displacement (together with single qubit and gaussian operations)

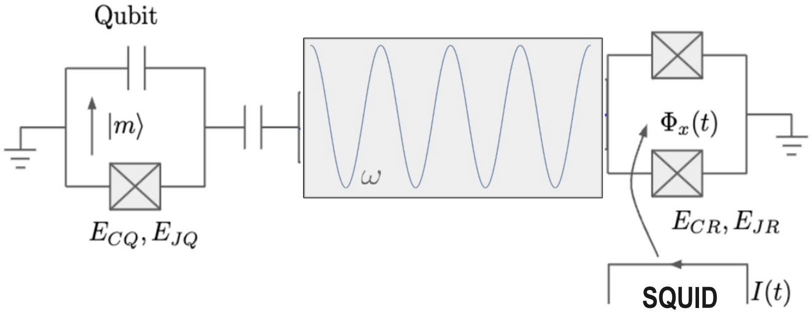
2. New Universal Gate: Controlled-Squeeze Gate

2. New Universal Gate: Controlled-Squeeze

SETUP

To implement the proposed controlled-squeeze gate we need to use three basic elements: one resonator and two circuits containing Josephson components located at each side

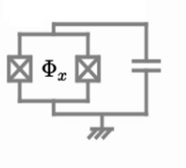
RESONATOR



The superconducting circuit at the right (SQUID) acts as a classical pump

The superconducting circuit at the left acts as a qubit with quantum states $|0\rangle$ and $|1\rangle$

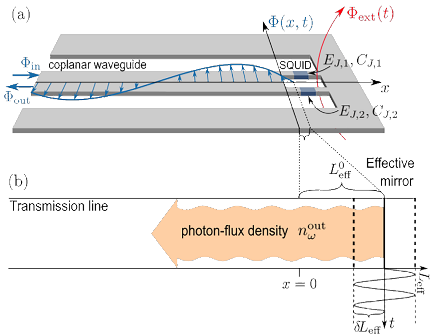
The resonator is terminated by a SQUID where we apply a time dependent flux

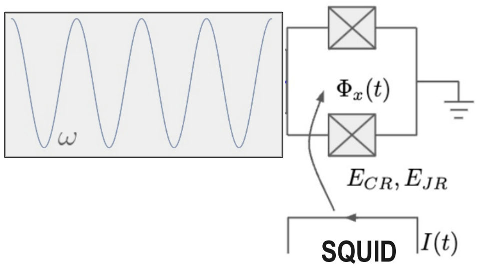


$$\Phi_x(t) = \epsilon \sin(w_d t)$$

2. New Universal Gate: Controlled-Squeeze

The lagrangian for a field inside a **superconducting resonator** of length d with inductance L0 and capacitance C0 per unit length, **terminated in a SQUID** at x = d





$\phi(t) = \Phi_x(t)$
External magnetic field flux

$$L = \left(\frac{1}{2e}\right)^2 \frac{C_0}{2} \int_0^d (\dot{\Phi}^2 - v^2 \Phi'^2) dx + \left(\frac{1}{2e}\right)^2 2C_J \int_0^d \frac{\dot{\Phi}^2}{2} \delta(x-d) dx + 2E_J \int_0^d \cos(\Phi) \cos\left(2e\phi(t)\right) \delta(x-d) dx.$$

Wustmann Shumeiko 2013

$v = 1/\sqrt{L_0 C_0}$

⋮

$$\ddot{\phi} - v^2 \phi'' = 0$$

$E_C = (2e)^2/(2C_J)$

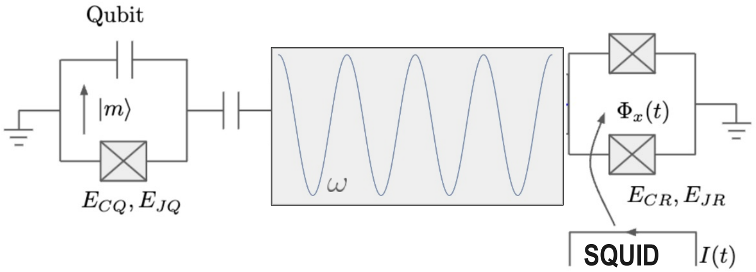
$E_{L,\rm cav} = (\hbar/2e)^2 (1/L_0 d)$

$$\hat{H}(t) = \omega \hat{a}^\dagger \hat{a} + g_d \epsilon \sin(\omega t - \theta)(\hat{a}^\dagger + \hat{a})^2$$

$$\frac{\hbar^2}{E_C} \ddot{\phi}_d + 2E_J \cos f(t) \phi_d + E_{L,\rm cav} d \phi'_d = 0,$$

2. New Universal Gate: Controlled-Squeeze

Hamiltonian of the model



$$\hat{H}(t) = \frac{\omega_q}{2}\hat{\sigma}_z + \omega \hat{a}^\dagger \hat{a} + \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z + g_d \epsilon \sin(\omega_d t - \theta)(\hat{a}^\dagger + \hat{a})^2,$$

Qubit

Dispersive coupling between the resonator and qubit

Coupling to external time-dependent flux

$$\omega_{0,1}(t) = \bar{\omega}_{0,1} + g_d \epsilon \sin(\omega_d t - \theta)$$

A state dependent resonator

$$|0\rangle \longrightarrow \bar{\omega}_0 = \omega + \chi$$

$$|1\rangle \longrightarrow \bar{\omega}_1 = \omega - \chi$$

Time dependent frequency squeezes the state of the resonator

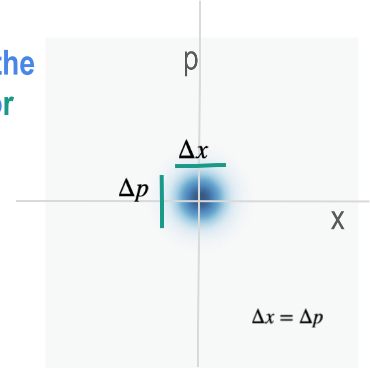
→ State dependences can be used to turn on and off the parametric resonance

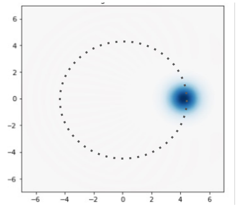
Controlled Squeeze Gate

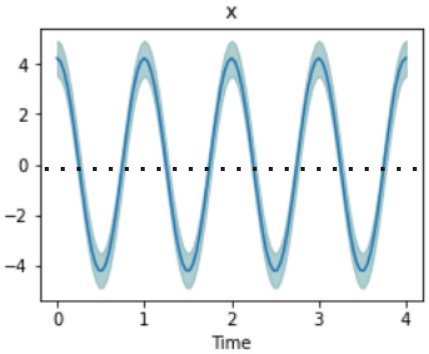
What is squeezing?

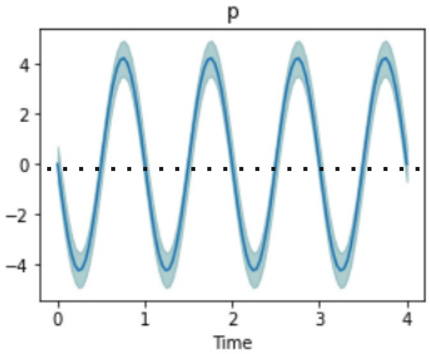
A coherent state has minimum uncertainty $\Delta x \Delta p = \frac{\hbar}{2} \rightarrow \hat{a}|\Psi\rangle = \mu|\Psi\rangle$

Coherent state of the harmonic oscillator

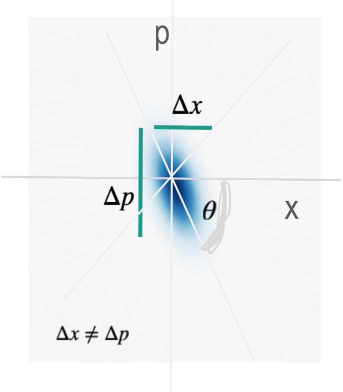






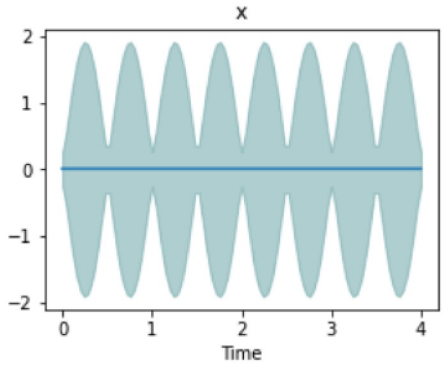


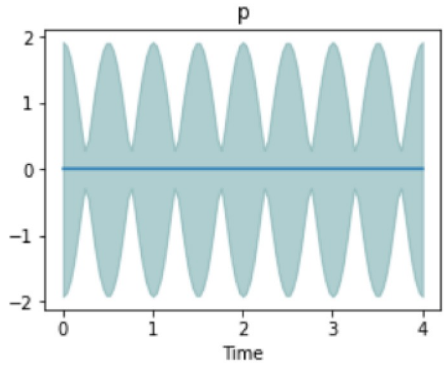
Squeezed state



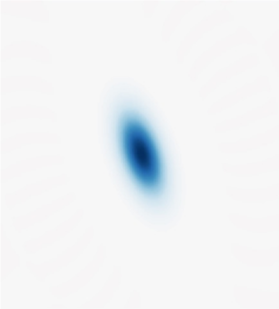
$$\Delta x = \sqrt{\frac{\hbar}{2m\omega}} e^{-r}$$

$$\Delta p = \sqrt{\frac{m\hbar\omega}{2}} e^{r}$$





What is Squeezing?



Squeezed state ⟶ eigenstate of $\hat{a}e^{i\theta}\cosh(r) + \hat{a}^{\dagger}e^{-i\theta}\sinh(r)$

$$|r,\theta\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{n=0}^{\infty}\left(\tanh(r)e^{i\theta}\right)^n\frac{\sqrt{2n!}}{n!}|2n\rangle$$

We define the squeezing operator $\hat{S}(r,\theta) = e^{\left(r/2(e^{-i\theta}\hat{a}^2 - e^{i\theta}\hat{a}^{\dagger 2})\right)}$

How to prepare a squeezed state?

Time dependence on the frequency induces squeezing

$$\hat{H}(t) = \frac{1}{2m}\hat{p}^2 + \frac{m}{2}\omega(t)^2\hat{x}^2$$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\frac{\hat{x}}{\sigma} + i\frac{\sigma}{\hbar}\hat{p}\right)$$

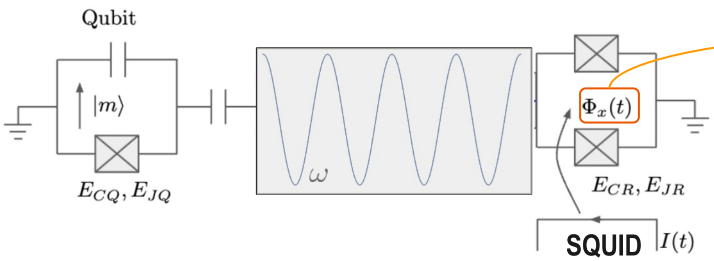
$$\dot{\hat{a}} = -i\omega\hat{a} - \frac{\dot{\omega}}{2\omega}\hat{a}^{\dagger}$$

$$a_t \rightarrow u\ a + v\ a^{\dagger}$$

$$|u_t|^2 - |v_t|^2 = 1$$

2. New Universal Gate: Controlled-Squeeze

Controlled Squeeze Gate Setup



We generate the squeezing by the external pumping of the SQUID, through parametric resonance

$$\omega_d = 2\bar{\omega}_1$$

From the Hamiltonian:

$$\hat{H}(t) = \frac{\omega_q}{2}\hat{\sigma}_z + \omega_r \hat{a}^\dagger \hat{a} + \chi \hat{a}^\dagger \hat{a} \hat{\sigma}_z + g_d \epsilon \sin(\omega_d t - \theta)(\hat{a}^\dagger + \hat{a})^2$$

$$e^{-i\hat{H}_I t} = \hat{S}(g_d \epsilon t, \theta) \otimes |1\rangle\langle 1| + \hat{U}_0(\tilde{\Delta} t) \otimes |0\rangle\langle 0|$$

In the Interaction representation and after the RWA

$$\hat{H}_I = \frac{i}{2} g_d \epsilon \left(e^{-i\theta}\, \hat{a}^2 - e^{i\theta}\, \hat{a}^{\dagger 2} \right) \otimes |1\rangle\langle 1| + \tilde{\Delta} \hat{a}^\dagger \hat{a} \otimes |0\rangle\langle 0|$$

$$\tilde{\Delta} = \bar{\omega}_1 - \bar{\omega}_0$$
$$\tilde{\Delta} \gg g_d \epsilon$$

If the control qubit is in state $|1\rangle$, the cavity field is squeezed

If the control qubit is in state $|0\rangle$, an harmonic evolution takes place (can be compensated)

Controlled Squeeze Gate

A universal gate

2. New Universal Gate: Controlled-Squeeze

$\hat{U}(t) := \quad \mathbf{C\text{-}Sqz}(r, \theta).$ → It applies a squeezing operation $\hat{S}(r,\theta)$ conditioned on the state of the qubit

Universal Gate It satisfied the following condition when combined with the Displacement operator

$$\hat{D}(\gamma)\text{:}\ \hat{S}(r,\theta)\hat{D}(\gamma)\hat{S}^{-1}(r,\theta) = \hat{D}(\gamma')$$

Which means that applying a displacement operator $\hat{D}(\gamma)$ in between two squeezing operators $\hat{S}^{-1}(r,\theta)$ y $\hat{S}(r,\theta)$ is equivalent to the application of a different displacement operator

The above relation among operators can be extended to control gates

$$\hat{D}^{-1}(\gamma)\mathbf{C\text{-}Sqz}(r,\theta)\hat{D}(\gamma)(\mathbf{C\text{-}Sqz})^{-1}(r,\theta) = \mathbf{C\text{-}Dsp}(\gamma' - \gamma)$$

The universality of $\mathbf{C\text{-}Dsp}(r,\theta)$ implies the universality of $\mathbf{C\text{-}Sqz}(r,\theta)$

2. New Universal Gate: Controlled-Squeeze

What does Universality mean?

Combined with:
$\mathrm{C-Sqz(r,}\theta)$

- Single qubit operations
- Gaussian operations in the resonators
- Qubit measurements

Can be used to create any quantum state of the qubit-resonator system

Controlled Squeezed Gate is universal if and only if Controlled Displacement Gate in universal

2. New Universal Gate: Controlled-Squeeze

A Controlled Squeeze Gate can be implemented with trapped ions 👇

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State-dependent motional squeezing of a trapped ion: Proposed method and applications

Martín Drechsler,[1] M. Belén Farías,[2] Nahuel Freitas,[2] Christian T. Schmiegelow,[1,*] and Juan Pablo Paz[1]
[1]*Departamento de Física, FCEyN, UBA and IFIBA, UBA CONICET, Pabellón 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*
[2]*Physics and Materials Science Research Unit, University of Luxembourg, Avenue de la Faváncerie 162a, L-1511 Luxembourg, Luxembourg*

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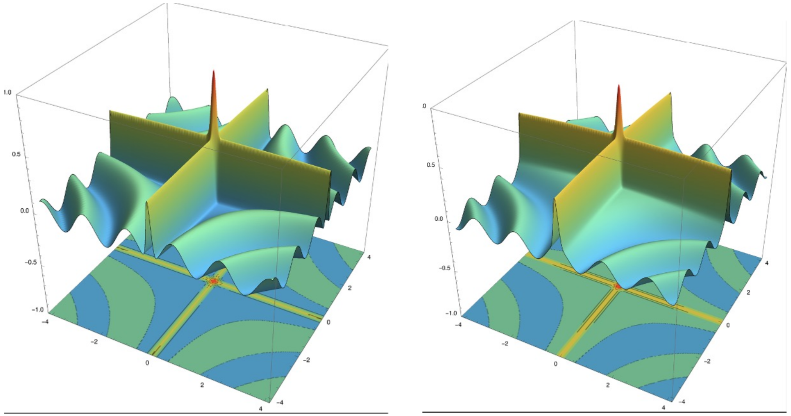
We show that the motion of a cold trapped ion can be squeezed by modulating the intensity of a phase-stable optical lattice placed inside the trap. The method we propose is reversible (unitary) and state selective: it effectively implements a controlled-squeeze gate. This resource could be useful for quantum information processing with continuous variables. We show that the controlled-squeeze gate can prepare coherent superpositions of states which are squeezed along complementary quadratures. Furthermore, we show that these states, which we denote "$\mathcal{X}$ states," exhibit a high sensitivity to small displacements along two complementary quadratures, which makes them useful for quantum metrology.

$$|\chi_\pm\rangle = \frac{1}{\sqrt{2c_\pm}}\left(|r,\tilde{\theta}\rangle \pm |r,\tilde{\theta}+\pi\rangle\right),$$

$|\chi_+\rangle$ $|\chi_-\rangle$



3. What can we do with a Controlled Squeeze Gate?

Encoding quantum states in the resonator

$$(\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle$$

INPUT

Encoding states of the resonator

OUTPUT

$$|0\rangle \otimes \left(\alpha|\chi_+\rangle + \beta|\chi_-\rangle\right)$$

Odd and even superpositions of squeezed states along two orthogonal directions in quadrature space

Have similar properties to the four-legged cat states as they are respectively superposition of **4*n* and 4*n* + 2 photon states**

This means that when losing a photon, the <u>error can be detected</u> by a subsequent parity measurement of the photon number inside the resonator

3. What can we do with a Controlled Squeeze Gate?

Protocol

$$|\Psi_Q\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\Psi_{QR}\rangle = \frac{1}{\sqrt{2}}\left(\alpha\, c_+|\chi_+\rangle + \beta\, c_-|\chi_-\rangle\right)|0\rangle + \frac{1}{\sqrt{2}}\left(\alpha\, c_-|\chi_-\rangle + \beta\, c_+|\chi_+\rangle\right)|1\rangle$$

State of the qubit

(i) Apply a Hadamard gate to the qubit

(ii) Apply a non-compensated $\mathrm{C-sqz}(r,\theta)$

(iii) Apply a π-rotation to the qubit

(iv) Apply the operator $\hat{U}(r, \theta + 2\varphi + \pi, \varphi)$

(v) Apply a π-rotation to the qubit

(vi) Apply another Hadamard gate to the qubit

$$c_\pm = \sqrt{1 \pm 1/\sqrt{\cosh(2r)}}$$

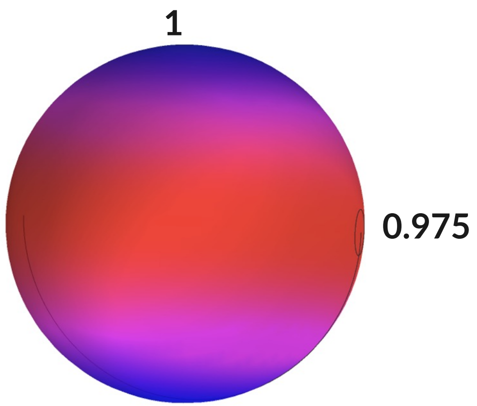
3. What can we do with a Controlled Squeeze Gate?

Fidelity

The fidelity is defined as $F = |\langle \Psi_{\text{ideal}} | \Psi_{\text{real}} \rangle|^2$

Mean Fidelity (maximum) $\bar{F} = \frac{1+P_z}{2} + \frac{1-P_z}{2}\sqrt{1-\frac{1}{\cosh(2r)}}$

$P_z = \alpha^2 - \beta^2$



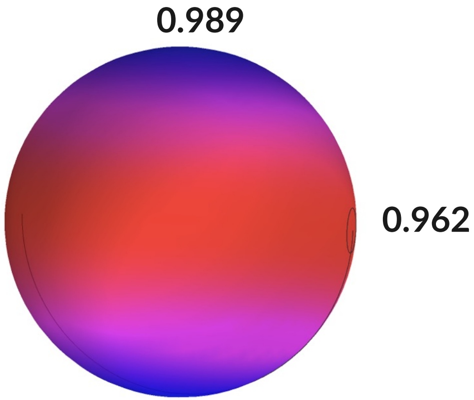
r=1.5

$\bar{F} \geq 0.995 \quad \rightarrow r \geq 2$

- We have simulated:

$\omega_r/(2\pi) = 6$ GHz
$\omega_q/(2\pi) = 4$ GHz
$\chi/(2\pi) = 8$ MHz
$g_d = 50$ MHz

Losses:
- Thermal Coupling 60 mk
- Relaxation time 200 micro s
- Gate time 200 ns



Purity values obtained are 97.3% (equator) and 99.3% poles

4. Summary

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- We presented a method for a universal quantum gate for Control Squeeze
- Parametric resonance is turned on and off by the state of the qubit
- Can be used for encoding quantum states in an error detectable way $\bar{F} \sim 1 - e^{-2r}\left(1 - P_z^2\right)/4$

A controlled-squeeze gate in superconducting quantum circuits

Nicolás F. Del Grosso,[1] Rodrigo G. Cortiñas,[2] Paula I. Villar,[1] Fernando C. Lombardo,[1] and Juan Pablo Paz[1]

[1] *Departamento de Física Juan José Giambiagi, FCEyN UBA and IFIBA UBA-CONICET, Facultad de Ciencias Exactas y Naturales, Ciudad Universitaria, Pabellón I, 1428 Buenos Aires, Argentina*

[2] *Department of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA*

We present a method to prepare non-classical states of the electromagnetic field in a microwave resonator. It is based on a controlled gate that applies a squeezing operation on a SQUID-terminated resonator conditioned on the state of a dispersively coupled qubit. This controlled-squeeze gate, when combined with Gaussian operations on the resonator, is universal. We explore the use of this tool to map an arbitrary qubit state into a superposition of squeezed states. In particular, we target a bosonic code with well-defined superparity and photon loss is thus error detectable by nondemolition parity measurements. We analyze the possibility of implementing this using state-of-the-art circuit QED tools and conclude that it is within reach of current technologies.